On the Foundations of Gravitation, Inertia and Gravitational Waves

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Abstract. After a hundred years of development of General Relativity, the derived cosmological models are requiring an ever increasing number of ad hoc corrections to be consistent with experimental observations. Because of the proven self consistency of General Relativity, it is possible that some of the principles involved in the foundations of General Relativity might not be valid under some circumstances. Foundations for gravitation and inertia can be derived from Maxwell equations and Special Relativity, which include new methods to analyze the anomalies of modern cosmology, the dynamics of space objects and gravitational wave propagation. With some restrictions, the results of this paper give the principles on which Newtonian gravity and General Relativity have been constructed, thus confirming most of the research made so far.

Keywords Euclidean Relativity, Special Relativity, General Relativity, Gravitational Waves, Maxwell Equations, Hyperspace.

1 Introduction

General Relativity has gained the status of most advanced, reliable and undisputed theory of gravity for a wide range of applications; among them it is possible to cite most of modern cosmology, gravitational wave experiments, frame dragging gyroscope experiments, down to more practical ones like spacecraft trajectory calculation, GPS atomic clock design and tuning, and many others. Unfortunately a handful of observational details seem nearly impossible to model using Einstein equations, moreover few other problems require the introduction of “entities” that it is not difficult to call unphysical, because they have never been directly detected in nature or produced in a laboratory, first among them the “dark matter” that seems to control the rotational dynamics of galaxies or the repulsive field that expands the universe right now.

It is not possible to negate that General Relativity is self consistent and very successful, but like all theories it is built on assumptions and principles. Till now these principles have been assumed to be absolutely valid under any circumstance and not derivable from a more fundamental “layer” of elementary physical phenomena.

The possibility of finding a more fundamental level of description for gravity may conditionally validate the fundamental principles of Newtonian gravity and General Relativity by redefining the domain of validity of those principles and also allow the understanding of those few details that cannot be included in the conventional analysis.

The more fundamental level of description must first produce Newtonian gravity and the principles related to inertia.

In this paper I propose to develop a more fundamental model based on Maxwell equations and the metric of Special Relativity.

The sole reason of this specific choice is that Maxwell equations have been validated by a large number of experiments in their usual fields of application; therefore, an extension to gravitation and cosmology seems an exciting possibility.

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[1] This document represents only the opinion of its author.

The underlying philosophical motivation is that Nature might want to choose a single set of computational mechanisms to evolve our reality, and after hundreds of years of laboratory experimentation we are pretty sure that Maxwell equations are the correct model for describing classical electromagnetism; therefore, after the adoption of Maxwell equations in three space dimension plus time for electromagnetism, we may want to adopt Maxwell equations in four space dimension plus time for gravity. Let's see how to proceed.
In historical order of formulation, I recall the model for the classical electromagnetic long range interaction, the Maxwell equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B_g} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}, \tag{4}$$

Later, the outcome of especially dedicated laboratory experiments on the propagation of electromagnetic waves was found to be in agreement with Lorentz invariance, in which the proper time (τ) distance between two events in space-time is invariant respect to a Lorentz transformation [15] and it depends on space-time coordinates according to the Minkowski metric:

$$(d\tau)^2 = c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2, \tag{5}$$

The Lorentz invariance in this paper it is simply interpreted as a relationship among differentials of motions along five coordinates; it is considered to be the outcome of experimentation and no additional physical interpretation is offered except that it shows that we have to deal with five coordinates, one of them (time t; $t>0$ in this formulation) depends on the remaining four.
By straightforward algebra, we have:

$$\left(\frac{d\tau}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = c^2, \tag{6}$$

where x, y, z, τ are defined to be the space coordinates of the so called Hyperspace. Because of the homogeneity suggested by the structure of equation (6), being $d\tau$ an absolute (i.e. invariant respect to a change in the reference system of coordinates) infinitesimal measure of distance along the τ coordinate, the same is assumed to be true for the remaining three coordinates x, y, z. Starting from a four dimensional origin, the Hyperspace is absolute, and being it characterized only by space like coordinates, it clearly has the properties of a memory space. This is a major difference respect to a space-time: points in a space-time are "events", points in the Hyperspace are "information" [6]. In fact if the coordinates of a n-dimensional space are all space-like, any function of the coordinates will define a property that will not change with time, it follows that this function represent a value memorized in a point that can be changed only by changing the function. Looking at equation (6), the Hyperspace is characterized by the universal four-speed c that is the speed of light.

2 Foundations

Maxwell equations are already applied to problems related to gravity, and nearly all modern approaches derive the Maxwell formulation of gravity from General Relativity, those are named Gravito-Electro-Magnetic (GEM) theories [3,11]. In spite of the recent surge in interest, the concept itself is older than General Relativity; see for instance Heaviside's paper [10], which is obviously unrelated to General Relativity.
The equations that we are here considering are proposed to become first principles for gravity and inertia. Following reference [8] I adopt the symbols of traditional GEM formulation with the fundamental difference that the present model is defined in five coordinates:

$$\nabla \cdot \mathbf{E_g} = -4\pi G \rho_g \, , \tag{7}$$

$$\nabla \cdot \mathbf{B_g} = 0 \, , \tag{8}$$

$$\nabla \times \mathbf{E_g} = -\frac{1}{c}\frac{\partial \mathbf{B_g}}{\partial t} \, , \tag{9}$$

$$\nabla \times \mathbf{B_g} = \frac{1}{c}\left(\frac{\partial \mathbf{E_g}}{\partial t} + 4\pi G \, \mathbf{J}_g\right) = \frac{1}{c}\left(\frac{\partial \mathbf{E_g}}{\partial t} + 4\pi G \rho_g \mathbf{v_\rho}\right), \tag{10}$$

where $\mathbf{E_g}$ and $\mathbf{B_g}$ are the gravitoelectric and gravitomagnetic fields respectively, $\mathbf{J_g}$ is the mass current density, ρ_g is mass density, G is the gravitational constant and c is the speed of light.

The signs in equation (10) are the same employed in equation (4), it means that mass-charges attract and co-aligned mass currents repel, with the result that a beam of mass-charges traveling at speed c in the Hyperspace does not collapse with time, therefore the choice to "stay" with Maxwell is motivated by stability reasons, which will later produce results consistent with observations. Differently, in the usual GEM theory, co-aligned mass-currents attract one another [3], it means that the gravitomagnetic force adds with the same sign to the gravitoelectric force, and the beam is more prone to collapse if it is relativistic in space-time.

In addition I add to the proposed formulation the proper time space dimension τ and the speed invariant of Euclidean relativity [9, 12, 4, 5]. Therefore, the new approach defines a four-dimensional space plus time Maxwellian theory of gravity that is directly applicable to masses in the Hyperspace domain.

At this level of abstraction it is necessary to be totally faithful to the proper Maxwell model for long range forces that we already know for electromagnetism, it means that all charges must be invariant under any static or dynamic condition. The straightforward consequence is that in the Hyperspace quantized mass-charges or massons must be the sources of the gravitational field. Therefore, particles that have mass and charge can be considered composed of sub-elementary particles that separately carry the mass-charge and the electric charge, they are named massons and chargeons, respectively. This composition is legitimate because the two properties of elementary particles that we are considering are invariant and independent, and in principle they could be separated. From the quark model it is known that chargeons have electric charge of + and – 1/3 e, which is invariant with respect to speed; the mass-charge of massons must also be invariant with respect to speed and it is supposed to be quantized on discrete values.

Regarding the speed invariant of equation (6), it is possible to introduce a four component refractive index of the Hyperspace. The new Maxwellian model of gravity for particles that obey Lorentz invariance is represented by equations (7) to equation (10) plus the following equation:

$$\mathbf{n}^2 \mathbf{v_\rho}^2 = c^2 \, . \tag{11}$$

Equation (11) indicates that all particles obeying Lorentz invariance must travel at some speed in the Hyperspace, the speed depends on the gravitational refractive index $\mathbf{n}$. Throughout this paper $\mathrm{n} = 1_{0,3}$; by this choice equation (11) represents Special Relativity [9].

3 The Gravitoelectromagnetic force in a special case

By adopting the formulation used for computing the electro-magnetic (EM) force at relativistic speeds between two moving charges with Maxwell equations (see [3], equations (8) and (9)), we find that the gravitoelectromagnetic force between two particles observed in space-time is:

$$F_r = F_{r\,gravitoelectric}\left(1 - \frac{v^2}{c^2}\right) = \frac{G}{r^2} m_1 m_2 \left(1 - \frac{v^2}{c^2}\right) \, , \tag{12}$$

where m_1 and m_2 are respectively the mass-charges of particle 1 and particle 2. Because of the additional complexity allowed by the four dimensional space domain, equation (12) is applicable only to particles that travel at the same speed along τ in Hyperspace, that are at the same proper time value (these points belong to a so called τ-frame) [5], that are at rest in x, y, z and are in different locations with distance r. To further simplify the discussion of the example we assume that $m_1 = m_2 = m$. Referring to equation (12), the first term in parentheses, unity, is the

gravitoelectric component, the second term is the gravitomagnetic component, v is the speed of the particles along the proper time coordinate.
In the Euclidean Relativity context from which the Hyperspace model is derived, all particles that are at rest in space-time, travel at the speed of light along the space coordinate τ, and according to equation (12) the gravitational interaction between two "massive" particles is zero ($v=c$) for any value of mass-charges considered. This is a puzzling result, because we do indeed observe the existence of the gravitational interaction in our space-time.
To overcome the apparent inconsistency, let's suppose that there exists an electromagnetic (there space dimensional) source of random motion that affects all charged sub-elementary particles (leptons and quarks); the source clearly is in the class of electromagnetic quantum fluctuations.
By recalling the speed invariant from the Minkowski metric expressed by equation (6), collecting the terms of speed in the three spatial coordinates of space-time $v_r^2 = v_x^2 + v_y^2 + v_z^2$ and the term in proper time, we have:

$$v_r^2 + v_\tau^2 = c^2 , \tag{13}$$

assuming that v_r of the two particles are reciprocally uncorrelated over a time interval, i.e. their motion is characterized by a random process with zero covariance, the related gravito-magnetic component of the force averages to zero over a time interval. Therefore, the force in space-time depends only on the fully correlated v_τ and becomes:

$$F_r = F_{r\,gravitoelectric}\left(1 - \frac{v_\tau^2(m)}{c^2}\right) = \frac{G}{r^2} m^2 \left(1 - \frac{(c^2 - v_r^2)}{c^2}\right), \tag{14}$$

$$F_r = \frac{G}{r^2} m^2 \left(v_r^2 / c^2\right). \tag{15}$$

If both particles have the same kinetic energy in space-time (including vibrational and rotational energy), that is required to keep them at the same value of τ, we have:

$$M^2 = m^2 v_r^2 / c^2 . \tag{16}$$

M is the mass of each particle in space-time. Its value has two factors, one is the mass-charge of the particle in Hyperspace, and the other is the square of the speed in the three-dimensional subset of space coordinates of Hyperspace related to a space-time. The presence of the two factors makes gravity different respect to other known forces in space-time.
To be consistent with the above formulation, it is necessary to assume that even if massons are responsible for the gravitational interaction in Hyperspace, they need to combine with chargeons to exhibit mass in space-time. In fact chargeons transfer their 3D zitterbewegung to the corresponding dimensional subset of the 4D massons to provide the observable dynamics that leads to Newton and Einstein gravitation.
Observable gravitational fields and masses of elementary particles depend on the masses of massons and on electromagnetic resonances of chargeons in the electromagnetic quantum fluctuations. Therefore, the stress-energy tensor, source of the gravitational field in space-time according to Einstein, does not describe pure energy; instead it describes "three dimensional" energy coupled to "four dimensional" massons. Electromagnetic energy that is not coupled to massons does not generate a gravitational field, which is the case for bare electromagnetic quantum fluctuations.
Under less restrictive conditions, gravitationally interacting particles might not have the same elementary mass m or the same kinetic energy. Therefore, their speed along the τ coordinate will be different: they will drift along τ, and gravity will be a fully four-dimensional interaction unless mass currents along τ are present [5], which may partially mask the experimental evidence in space-time by a hyper-cylindrical symmetry.
A special case is photons, for which $v_r = c$ and $v_\tau=0$ (see equation 6) in our space-time. Therefore, photons are allowed to have Hyperspace mass m and have its effect made unobservable by interactions of co-moving photons in a beam.
Photon mass m, if present, can only be observed by the interaction between photons and a gravitational field originated by matter or by crossing/colliding photon beams or a combination of these effects (for instance fiber laser gyro based experiments), see equation (23); equation (6) may also point to possible parity violations in space-time, see for instance [13].
Because photons are characterized by $v_\tau=0$ and matter exhibit $v_r \approx 0$, because of equation (6) it is deduced that photons do indeed transfer information between universes in the layered structure of the multiverse [5].

4 Speed composition applied to nucleons

In this section the Maxwell formulation of gravity is applied to few simple problems to understand if predictable effects may find confirmation with real experiments.
Equation (16) indicates that M depends on motion and vibrations of particles. It is possible to write a speed composition with large-scale speeds in the three-dimensional subset of Hyperspace and take into account that the average (unsigned) internal speed is almost constant:

$$v_r = \left(v_{\text{int}}^2 + v_{ext}^2\right)^{1/2} . \tag{17}$$

For instance it is known that quarks have relativistic speed inside nucleons, for them its is possible to write $v_{\text{int}}^2 >> v_{ext}^2$. Therefore, assuming that v_{int}, v_{ext} are modeled by a random process with zero covariance:

$$v_r = \left(v_{\text{int}}^2 + v_{ext}^2\right)^{1/2} = v_{\text{int}}\left(1 + \frac{v_{ext}^2}{v_{\text{int}}^2}\right)^{1/2} \cong v_{\text{int}}\left(1 + \frac{1}{2}\frac{v_{ext}^2}{v_{\text{int}}^2}\right), \tag{18}$$

$$M \cong m\frac{v_{\text{int}}}{c}\left(1 + \frac{1}{2}\frac{v_{ext}^2}{v_{\text{int}}^2}\right). \tag{19}$$

Equation (19) is the expression for the usual rest mass. If the speed of internal motion tends to c then space-time mass M tends to Hyperspace mass m. Respect to plain Newtonian gravity, rest mass M has an additional component that depends on the kinetic energy related to external speed and the correlation assumption of equation (14). A major departure from the standard relativistic approach is that v_{ext} is absolute in the three dimensional subset of coordinates of the Hyperspace, it follows that, depending on the motion of an object, let's say the earth, respect to any large scale reference system in which the earth may have a dominant speed, the mass M of the earth may slightly change during its orbital motion. Equation (19) might be employed to investigate various interplanetary spacecraft flybys anomalies already observed [1]. According to the references these anomalies cannot be modeled with General Relativity and seem to depend on the velocity vector of the spacecraft respect to the ecliptic during the planetary fly-by. With available data, the order of magnitude of the phenomenon seems compatible with a mass change during fly-by as expressed by equation (19). If this approach will be proven correct, this model could be validated.
It must be noted that the well known relativistic mass increase is due to "relative" motion in Hyperspace, it is a consequence of equation (5), that is one of the fundamental hypotheses of this model, and it must not be confused with the result of equation (19). In fact the speed in proper time is:

$$v_\tau^2 = c^2 - v_r^2; \qquad d\tau = dt\left(\frac{c^2 - v_r^2}{c^2}\right)^{1/2}, \tag{20}$$

and for relativistic speeds $v_r \approx< c$ we have, respect to non relativistic ($d\tau \approx dt$) observers in space-time:

$$Mdt = Md\tau\left(\frac{c^2}{c^2 - v_r^2}\right)^{1/2}. \tag{21}$$

It can be extended to objects and observers with different v_r, both different from zero.

5 Conservation of momentum

After the relevant result of the description of the Coulomb like gravitational force in space-time as a function of Hyperspace mass and electromagnetic energy, it is possible to investigate how quarks and leptons dynamically interact because of gravitomagnetic coupling in Hyperspace. According to [5], Euclidean relativity implies layered space-times structures in the Hyperspace (a multiverse with layered space-times), this leads to currents of particles, with a dominant component for matter particles directed along the τ coordinate.
The Faraday-Henry law of equation (3) has been adopted to describe the working principles of electrical transformers for alternate current (AC); it has a gravitational counterpart via equation (9).

Let's take two ring shaped flows of particles in which a small section along the ring approximate the flow of two interacting space-time particles and their histories in the multiverse. In electrodynamics this is a 1:1 current transformer operating in the short circuit configuration, and with unitary coupling constant *k* (see [4] for a detailed calculation of the relationship between *k* and the geometry of the currents in an AC electrical transformer) we have for changes in currents:

$$\Delta I_1 = -\Delta I_2 \,. \tag{22}$$

This equation is valid even if there is a common direct current (DC) flowing in both rings (i. e. the speed invariant of equation (6)); result that has been already proven in electrodynamics and that can be observed in all electrical transformers. Interestingly, in this model the DC component is the origin of the gravitational force, the varying component (the analogue of the AC component of electrodynamics) is now discussed. Expressing the current as speed times mass-charge per unitary length μ we have:

$$\Delta v_1 \mu_1 = -\Delta v_2 \mu_2 \tag{23}$$

that, decomposed in the four orthogonal components, is conservation of momentum for Newton mechanics.
It is relevant to understand that, because conservation of momentum is an experimental fact, in the present model conservation of momentum is entitled to confirm the existence of the Hyperspace and its layered space-times structures (branes of the multiverse) and the existence of mass currents.
The four orthogonal components of equation (23) must be combined with equation (6), one for each particle, to define the equations of motion.

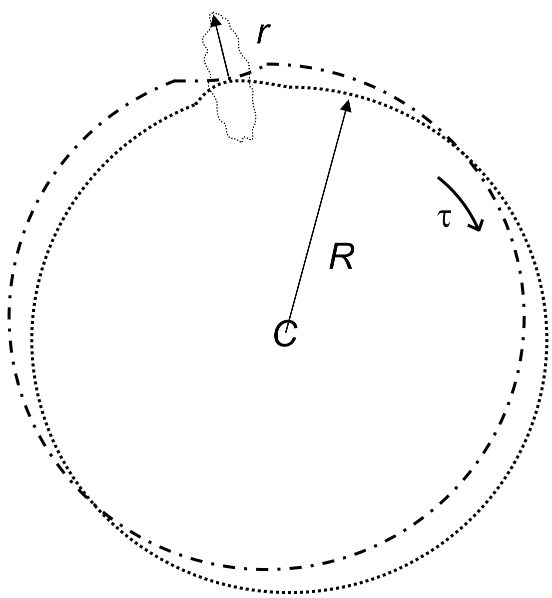

Figure 1. Conservation of momentum implies the existence of coupled mass currents in Hyperspace.

Figure 1 depicts the simplest structure of coupled hyperspace mass currents that are required to originate conservation of momentum. Small *r* can be interpreted as the actual radius of the universe in three dimensions, *R* is the radius of the four dimensional universe that includes parallel space-time universes. The overall shape of the four dimensional universe is annular or toroidal as a prerequisite to have high coupling factor and the validity of conservation of momentum [4].

It is also possible to speculate that there may exist a focus point along the τ coordinate, where all currents merge, this point may represent the classical "big bang" for this specific model.

6 Newton Dynamics

The most famous Newton law is considered to be the result of mathematical modeling of experimental observations. It is indeed possible to deduce it form gravitomagnetic coupling under the restrictive condition of perfect coupling, which is always valid at small and planetary scale and far from *C* as depicted in Figure 1 at cosmological scales.
From equation (23), after integration of mass density over any suitable length, infinitesimal changes in speeds in an isolated system respect to a common time give:

$$F = a_1 m_1 = -a_2 m_2 , \qquad (24)$$

that defines a long range force *F,* which is entitled to mediate conservation of momentum.
It is important to observe that the present approach treats mass like an electric charge and no additional particle dynamics can be imposed to the model except for the one derived from Maxwell equations. Therefore, the model can only describe stable or metastable hyperspace particle current configurations that, as soon as they are sampled by a space-time brane, the experimental reality (with its Newtonian dynamics) appears. These currents must be induced, guided and stabilized by superimposed gravitoelectromagnetic fields and, if not for non-gravitational interactions, they would describe a static Hyperspace.
Referring to Figure 1, because the validity of conservation of momentum is limited to distances between the coupled Hyperspace mass currents much smaller than *R*, if the observable space-time universe extends to the center *C* of the four dimensional universe, near *C* it will be possible to observe violations to Newton dynamics and Kepler laws down to the planetary scale.

7 Gravitational waves

It is well known that Maxwell equations have wave solutions in a vacuum, i.e. very far from any source of the fields. It is also very well known that these solutions match observation with extremely high accuracy for classical electromagnetic systems.
The same analysis is here proposed to be applied to Gravitoelectromagnetic Maxwell equations in the Hyperspace. Gravitoelectromagnetic waves are those waves that are produced by the motion of massons in Hyperspace. If massons are part of known particles the motion of these specific massons is constrained in the three dimensional brane, but gravitoelectromagnetic wave propagation is not.
For this model the gravitoelectromagnetic waves are "gravitational" waves (GWs). Because of the present embryonic stage of development of this model, they may or may not be comparable or fully related to GWs as described by the theory of General Relativity (GWs of General Relativity).
From Equations 3 to 6, substituting $E'_g = E_g\, c$, adopting the usual methodology employed for the derivation of wave equations for electromagnetism, we find the corresponding equations for the gravitomagnetic waves and the gravitoelectric waves.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E'_g = 0 ; \qquad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)B_g = 0 \qquad (25)$$

It must be observed that time *t* must be interpreted in a pure classical non-relativistic sense, i.e. it is an absolute time. Despite the very familiar look, equations (25) have solutions that differ from the electromagnetic case. The difference is due to dimensionality and the structure of the sources in Hyperspace.
In hyperspherical coordinates the equations can be solved by separation of variables. Let's refer to one of them, being the other one formally equivalent. Put $B_g(r,t) = T(t)\, R(r)$ and i=4 the individual equations are [7, 2]:

$$r\frac{d^2R}{dr^2} + (i-1)\frac{dR}{dr} + k^2 rR = 0 ; \qquad \frac{d^2T}{dt^2} + \omega^2 T = 0 , \qquad (26)$$

The second equation (26) is the temporal part, whose solution is trivial. The first equation (26) is the spatial part, whose solution can be written in terms of harmonic functions if *i* is odd. If *i* is even the solutions are no longer harmonic. Assuming that R(r) can be expressed with the series:

$$R(r) = c_0 r^q + c_1 r^{q+1} + c_2 r^{q+2} + \ldots \tag{27}$$

The coefficient c_{2j} for the solution with i=4 is given by:

$$c_{2j} = c_0 \frac{\left(-k^2\right)^j}{2^{2j} j!(1+j)!} \tag{28}$$

Therefore:

$$R(r) = c_0\left[1 - \frac{k^2 r^2}{8} + \frac{k^4 r^4}{192} - \frac{k^6 r^6}{9216} + \ldots\right] = \frac{2 c_0 J_1(kr)}{kr}, \tag{29}$$

where J_1 is the first order Bessel function.
Equation (29) must be compared to the solution of the spatial part for waves in three dimensions, i.e. electromagnetic waves, that is:

$$R(r) = \frac{c_0 \sin(kr)}{kr} \tag{30}$$

Therefore, GWs exhibit an additional attenuation of the amplitude of the wave with the distance from the source respect to the more usual propagation of waves in three dimensions, as for instance electromagnetic waves and GWs of General Relativity. The spatial frequency of the wave tends to stabilize with distance because the zeroes of $J_1(x)$ are separate by π for large x.
From the energy conservation standpoint it is possible to compare electromagnetic and gravitational waves: The propagation of electromagnetic wave-packets or pulses in three dimensional space plus time (space-time) can be easily expressed in terms of the integration of energy over a finite expanding shell because the waves in the packet travel at a single speed. In fact, solutions of the second of equations (26) and $R(r)$ of equation (30) are both written in terms of sine and cosine functions.
GWs propagation in four dimensions plus time is characterized by a frequency spectrum in time domain that is preserved during propagation (temporal part) like for electromagnetic waves, instead wavelength depends on distance because of equation (29).
Therefore, in the Hyperspace, wave-packets are characterized by an edge of the wave and a trail or afterglow that follows the edge with propagation speeds distributed between a maximum and zero. If a source emits a, say, millisecond burst that may define an expanding millisecond shell in four space dimensions plus time, after a given propagation time, the energy content of the shell is less than the initial content, in fact it is always necessary to integrate energy over the whole four-dimensional volume within the shell external surface to obtain a constant.
If the source of GWs is monochromatic, then equation (29) is an expression of the amplitude of the wave as a function of the radial distance from the source. It is possible to adopt an asymptotic expansion of J_1 [14] in order to approximate the amplitude envelope of the spatial part of the wave.

$$Env\{R(r)\} = \frac{2 c_0 Env\{J_1(kr)\}}{kr} \approx c_0 \frac{2}{kr}\sqrt{\frac{2}{\pi kr}};\ r > 0 \tag{31}$$

That clearly describes the effect of the additional large dimension, adopted in the present GW model, on the amplitude of a propagating periodic wave. The propagation law is consistent with the analysis reported by ref [2] because of the manifold dimensionality here adopted.

8 Conclusion

A new model for the gravitational interactions based on Maxwell equations in four space dimensions plus time and Special Relativity has been proposed. The model is a radically new approach.
Under most circumstances the results of this model produce the principles that allowed the foundations of Newtonian gravity and General Relativity, small discrepancies have been found that may help modeling and understanding astronomical observation that defy the straightforward analysis based on current theories.

Gravitational waves have been studied, and propagation laws were found to be in agreement with a recently published analysis based on the linearized Euclidean formulation of General Relativity.

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