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PROPER-TIME CLASSICAL ELECTRODYNAMICS

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We construct a generalization of Maxwell's equations associated with the proper-time of the source which accounts for radiation reaction without any assumptions concerning the nature of the source. The theory leads to a new invariance group, related to the Lorentz group, which leaves the proper-time of the source fixed for all observers.

Key words: special relativity, proper time, radiation reaction.

1. INTRODUCTION

It was observed in [1] that the use of time as a fourth coordinate (as introduced by Minkowski [2]) in the special theory of relativity is a third postulate, distinct from the first two:

(1) The physical laws of nature and the results of all experiments are independent of the inertial frame of the observer.
(2) The speed of light (relative to all inertial observers) is constant.

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This is, of course, independent of time being a fourth dimension.

As Minkowski was a well-known mathematician, this postulate was embraced by many. Others including Einstein, Lorentz, and Poincaré - regarded it as a mathematical abstraction lacking physical content and maintained that space and time have distinct physical properties. Although Einstein demurred, the feeling among many of the other leading physicists at the time was that an alternate implementation should be possible which preserved some remnant of an "absolute time" variable. The inability to obtain a viable alternative forced acceptance of the current implementation.

By convention, the proper time used to describe physical systems is that of each observer. Since it changes from observer to observer, it is not an invariant concept and there is no general way to choose a fixed observer clock. Horwitz and Piron [6] solve this problem by postulating a fifth (nonobservable) time evolution parameter which represents a universal (or historical) clock which is in principle available to all observers. Fanchi defines a possible measurement process for this variable at the quantum level (see his book [16] and references therein).

In previous papers [1,3-4], we have constructed an alternate implementation of the first two postulates of special relativity without assuming that time be treated as a fourth coordinate (although it can be). Our approach is based on the observation that we may use the unique proper-time associated with the system of physical interest in place of the observer proper-time. The use of such a variable is not new and dates back to Tetrode and Fock (for a recent review see Fanchi [5]). Our approach in [1] is close that of Horwitz and Piron [6], but distinct in that the system proper-time is an observable representing a possible clock available to all experimenters in their frame of reference. As such, we treat the transformation from observer proper-time to system proper-time as a canonical (contact) transformation on extended phase space. This approach forces the identification of the canonical Hamiltonian which generates the Lie algebra (Poisson) bracket and thereby leads to a conceptually and technically much simpler implementation of the special theory.

In this paper, we construct a direct representation of the group associated with proper-time transformations between observers for a given system. This allows us to construct a proper-time generalization of Maxwell's equations which has a new invariance group leaving the proper-time of the source fixed for all observers. The new group is closely related to, but distinct from, the Lorentz group. It was shown in [1] that, in the free particle case, this group generates a similarity transformation of the Poincaré group by the canonical proper-time group.

When we construct the associated wave equation, a damping term appears. This term is of the correct type and order of magnitude to account for the observed radiation reaction in classical electrodynamics. It occurs instantaneously with acceleration and can be positive or negative depending on the forces. Our theory is not invariant under time inversion, we thus lead to associate retarded solutions with matter while advanced solutions are associated with anti-matter. This leads to a natural arrow for (proper) time.

2. BACKGROUND

In order to make things transparent, we follow the original approach of Einstein [7]. Let us consider two inertial observers X and X', with X' moving along the positive x-axis with velocity v as seen by X. Let a particle (the source of an electromagnetic field) also move along the x- axes with velocity w_x as seen by X and velocity $w'_{x'}$ as seen by X'. We also assume that the (proper) clocks of X and X' both begin when their origins coincide (Einstein synchronization). It follows as in [7], that:

$$\begin{aligned} x' &= \gamma(v)(x - vt), \;\; y' = y, \;\; z' = z, \;\; t' = \gamma(v)(t - vx/c^2); \\ x &= \gamma(v)(x' + vt'), \; y' = y, \;\; z' = z, \;\; t = \gamma(v)(t' + vx'/c^2), \end{aligned} \tag{1}$$

with $\gamma(v) = [1 - v^2/c^2]^{-1/2}$, represent the Lorentz transformations between our two observers X and X'.

The two observers X and X' can compute the proper-time for the source in three ways. The first approach, due to Minkowski, is well-known: $\gamma(w)dt = d\tau$ and $\gamma(w')dt' = d\tau$. The second approach (used in [1]) is based on the fact that the Hamiltonians

$$H = mc^2[1 - w^2/c^2]^{-1/2}, \quad H' = mc^2[1 - w'^2/c^2]^{-1/2} \tag{2}$$

so that

$$d\tau = \frac{mc^2}{H}dt = \frac{mc^2}{H'}dt'. \tag{3}$$

In the third case, we use

$$dt = \delta(u)d\tau, \quad dt' = \delta(u')d\tau, \quad \delta(u) = [1 + u^2/c^2]^{1/2}, \tag{4}$$

where $u_x = dx/d\tau$ is the (proper) velocity of the source as seen by X and $u'_{x'} = dx'/d\tau$ is the source velocity as seen in the X' system. This relationship is easy to derive using

$$u = w[1 - w^2/c^2]^{-1/2}, \tag{5a}$$

and, solving for w in terms of u,

$$\begin{aligned} w &= u[1 + u^2/c^2]^{-1/2}, \\ [1 - w^2/c^2]^{-1/2} &= [1 + u^2/c^2]^{1/2}. \end{aligned} \tag{5b}$$

3. PARTICLE DYNAMICS

For the dynamics of any classical observable W associated with the particle, the Poisson bracket defines Hamilton's equations in the X frame by

$$\frac{dW}{dt} = \frac{\partial H}{\partial p}\,\frac{\partial W}{\partial x} - \frac{\partial H}{\partial x}\,\frac{\partial W}{\partial p} = \{H, W\}, \tag{6a}$$

so that

$$\dot{x} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x}. \tag{6b}$$

Next, using $dt = H/mc^2 d\tau$, the time evolution of the function W is given by the chain rule

$$\frac{dW}{d\tau} = \frac{dW}{dt}\,\frac{dt}{d\tau} = \frac{H}{mc^2}\{H, W\}. \tag{7}$$

The energy functional K conjugate to the proper-time τ must therefore satisfy

$$\{K, W\} = \frac{H}{mc^2}\{H, W\}. \tag{8}$$

In the free-particle case, $H = [m^2c^4 + c^2p^2]^{1/2}$, and since m remains invariant during the dynamics, the form of the functional K can be directly determined from

$$\begin{aligned} \frac{H}{mc^2}\{H, W\} &= \frac{H}{mc^2}\,\frac{\partial H}{\partial p}\,\frac{\partial W}{\partial x} - \frac{H}{mc^2}\,\frac{\partial H}{\partial x}\,\frac{\partial W}{\partial p} \\ &= \frac{\partial}{\partial p}\left(\frac{H^2}{2mc^2} + a\right)\frac{\partial W}{\partial x} - \frac{\partial}{\partial x}\left(\frac{H^2}{2mc^2} + a'\right)\frac{\partial W}{\partial p}, \end{aligned}$$

where a, a' are arbtrary constants. In the zero - momentum frame ($\mathbf{p} = 0$), we have $H = mc^2$, so that $K = mc^2$ (from $\{K, W\} = H/mc^2\{H, W\}$). It follows that $a = a'$ and

$$K = \frac{H^2}{2mc^2} + \frac{mc^2}{2}, \quad \frac{dW}{d\tau} = \{K, W\}. \tag{9}$$

In a similar manner, the observer in X' will obtain

$$K' = \frac{H'^2}{2mc^2} + \frac{mc^2}{2}, \quad \frac{dW'}{d\tau} = \{K', W'\}. \tag{10}$$

This physically intuitive approach provides insight but is far from rigorous. In fact, the above approach does not insure that we have a canonical transformation of variables. It is easy to check that:

$$\mathbf{p} \cdot d\mathbf{x} - H dt = \mathbf{p} \cdot d\mathbf{x} - K d\tau + dS, \tag{11}$$

where $dS = (1/2)(mc^2 - H^2/mc^2)d\tau$, so that the above transformation is canonical. The general solution to the problem is solved by:

$$K = mc^2 + \int_{mc^2}^{H} (dt/d\tau) dH' = mc^2 + \int_{mc^2}^{H} (H'/mc^2) dH'. \tag{12}$$

In [1], we showed that the above considerations generate a similarity action on the Poincaré group and *thus proves that the laws of physics will be the same for all observers. In the many-particle case, we proved the following results in [4]:*

Theorem 1. There exists a unique observable clock for the time-evolution of any closed interacting relativistic system.

Theorem 2. There is a many - particle direct - interaction theory with the following properties:

1. The theory satisfies the first two postulates of special relativity.
2. The theory is based on Hamiltonian dynamics.
3. The theory is based on independent (canonical) particle variables.

It is known that replacement of the first condition with the requirement of Lorentz invariance is only compatible with noninteracting particles. This is the content of the no-interaction theorem (see [1] and references therein).

4. TIME REVERSAL NONINVARIANCE

Since $d\tau = (mc^2/H)dt$, $K = [H^2/2mc^2 + mc^2/2]$, and m are always positive, we see that, if $t \to -t$ (time reversal) or $H \to -H$, then

$K \to K$ is invariant, while $\tau \to -\tau$. *Thus our theory is noninvariant under time reversal at the classical level and since τ is monotonically increasing, we acquire an arrow for (proper) time.* It is thus natural to interpret anti-matter as matter with it's proper time reversed. A complete discussion requires the introduction of Santilli's isodual numbers [15], in which the unit 1 is replaced by -1 and $ab \to a * b = -ab$ so that $(-1)*(-1) = -1$. This allows for a completely symmetric theory of matter (and numbers) which avoids all of the objections to hole theory, while maintaining consistency with our physical sense of a monotonically increasing of time variable. We will discuss this completely as a part of our approach to the foundations of relativistic quantum theory. The arrow of historical time has been discussed extensively by Fanchi in his book [16] (see also [17]) and by Horwitz, et al.[18].

Both Feynman [8] and Stueckelberg [9] introduced the notion of representing anti-matter as matter with its time reversed. Our final conclusion is the same as theirs however, the two approaches are distinct. In our approach, we replace t by τ and aquire K as its cannonical Hamiltonian, so that τ becomes both our coordinate time and evolution parameter. In their approach, they retain t and H and introduce τ as an evolution parameter. This allows them let $d/d\tau$ maintain the role of a Lorentz invariant (operator-valued) quantity.

5. PROPER-TIME TRANSFORMATION

In order to construct an explicit representation for the above transformation, we use our third representation for the proper time. Let us return to (1) and note that $\mathbf{w}$ and $\mathbf{w}'$ are related to $\mathbf{u}$ and $\mathbf{u}'$ by

$$\begin{aligned} \mathbf{u}' &= \gamma(w')\mathbf{w}', \quad \mathbf{w}' = \delta(u')^{-1}\mathbf{u}', \\ \mathbf{u} &= \gamma(w)\mathbf{w}, \quad \mathbf{w} = \delta(u)^{-1}\mathbf{u}. \end{aligned} \tag{13}$$

From

$$w'_{x'} = \frac{(w_x - \mathbf{v})}{(1 - vw_x/c^2)}, \qquad w'_{y'} = \frac{w_y}{\gamma(v)(1 - vw_x/c^2)}, \tag{14a}$$

$$w'_{z'} = \frac{w_z}{\gamma(v)(1 - vw_x/c^2)}, \tag{14b}$$

we have

$$\begin{aligned} \delta(u')^{-1}u'_{x'} &= \frac{u_x - \delta(u)v}{\delta(u)[1 - (vu_x/\delta(u)c^2)]}, \\ \delta(u')^{-1}u'_{y'} &= \frac{u_y}{\gamma(v)\delta(u)[1 - (vu_x/\delta(u)c^2)]}, \\ \delta(u')^{-1}u'_{z'} &= \frac{u_z}{\gamma(v)\delta(u)[1 - (vu_x/\delta(u)c^2)]}. \end{aligned} \tag{14c}$$

If $\mathbf{w}$ is constant, then $\mathbf{u}$ is constant and we get, from (4) and (13), $t = \delta(u)\tau$ and $t' = \delta(u')\tau$, so that

$$x' = \gamma(v)(x - v\delta(u)\tau), \quad x = \gamma(v)(x' + v\delta(u')\tau). \tag{15a}$$

In the general case, $\mathbf{w}$ is not constant, so that

$$t = \int_0^\tau \delta[u(s)]ds, \quad t' = \int_0^\tau \delta[u'(s)]ds. \tag{16}$$

It follows that t (or t') is nonlocal as a function of τ in the sense that the value depends on the particular physical history (proper time path) of the source. Setting

$$\Lambda(h) = 1/\tau \int_0^\tau \delta[h(s)]ds, \tag{17}$$

our transformations between observers become:

$$\begin{aligned} &x' = \gamma(v)(x - v\Lambda(u)\tau), \quad x = \gamma(v)(x' + v\Lambda(u')\tau), \\ &u' = \gamma(v)(u - v\delta(u)), \quad u = \gamma(v)(u' + v\delta(u')), \\ &a' = \gamma(v)[a - (v/c^2)\mathbf{u}\cdot\mathbf{a}\delta(u)^{-1}], \\ &\ a = \gamma(v)[a' + v/c^2)\mathbf{u}'\cdot\mathbf{a}'\delta(u')^{-1}]. \end{aligned} \tag{15b}$$

where $a(a')$ is the particle proper (-three) acceleration.

It should also be noted that, by the mean value property for integrals, we can find a unique $s(\tau)$ for each τ, with $0 < s(\tau) < \tau$ such that $u_\tau = u(\tau - s(\tau))$ and $\Lambda(u) = \delta(u_\tau)$. It is clear that this property is observer independent since $t' = \gamma(v)[t - (vx)/c^2] \Rightarrow \Lambda(u') = [\Lambda(u) - (vx)/c^2]$.

If another observer is also present, then using $x'' = \gamma(v')(x' - v'\Lambda(u')\tau)$ and (15), we easily obtain:

$$x'' = \gamma(v')\gamma(v)\left(1 + vv'/c^2\right)\left[x - \frac{v + v'}{(1 + vv'/c^2)}\Lambda(u)\tau\right], \tag{18a}$$

$$\gamma(v'') = \gamma(v')\gamma(v)(1 + vv'/c^2). \tag{18b}$$

It follows that we get the same (Lorentz) velocity addition law for (inertial) observers:

$$v'' = \frac{v + v'}{(1 + vv'/c^2)}. \tag{19}$$

6. PROPER-TIME MAXWELL EQUATIONS

From (3) and (15), it is easy to see that:

$$\begin{aligned} \partial_t &= (1 + u^2/c^2)^{-1/2}\partial_\tau, \\ \partial_t &= \gamma(v)(\delta(u')^{-1}\partial_\tau - v\partial_{\mathbf{x}'}), \\ \partial_{\mathbf{x}} &= \gamma(v)(\partial_{\mathbf{x}'} - (v/c^2)\delta(u')^{-1}\partial_\tau); \end{aligned} \tag{20a}$$

$$\begin{aligned} \partial_{t'} &= (1 + u'^2/c^2)^{-1/2}\partial_\tau, \\ \partial_{t'} &= \gamma(v)(\delta(u)^{-1}\partial_\tau + v\partial_{\mathbf{x}}), \\ \partial_{\mathbf{x}'} &= \gamma(v)(\partial_{\mathbf{x}} + (v/c^2)\delta(u)^{-1}\partial_\tau). \end{aligned} \tag{20b}$$

If we set $b^2 = (u^2 + c^2)$, then we have $1/c\partial_t = 1/b\partial\tau$ and $1/c\partial_{t'} = 1/b\partial\tau$.

Let us now consider Maxwell's equations as seen from X for the field at the source. We assume that the current density $\mathbf{J}$ can be written as $c\mathbf{J} = \rho\mathbf{w}$. Using (13), (15) and (20), these equations can be written as *(generalized to depend on the proper-time of the source)*:

$$\begin{aligned} &\nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{E} = 4\pi\rho, \\ &\nabla \mathrm{x} \mathbf{E} = -\frac{1\partial\mathbf{B}}{b\partial\tau}, \quad \nabla \mathrm{x} \mathbf{B} = \frac{1}{b}\left[\frac{\partial\mathbf{E}}{\partial\tau} + 4\pi\rho\mathbf{u}\right]. \end{aligned} \tag{21}$$

Following the same calculations as in Einstein [7], we find that:

$$\begin{aligned} E'_{x'} &= E_x, & B'_{x'} &= B_x, \\ E'_{y'} &= \gamma(E_y - v/cB_z), & B'_{y'} &= \gamma(B_y + v/cE_z), \\ E'_{z'} &= \gamma(E_z + v/cB_y), & B'_{z'} &= \gamma(B_z - v/cE_y), \\ \rho' &= \rho(1 - vu/bc), \end{aligned} \tag{22}$$

which lead to:

$$\begin{aligned} &\nabla' \cdot \mathbf{B}' = 0, \quad \nabla' \cdot \mathbf{E}' = 4\pi\rho', \\ &\nabla' \mathrm{x} \mathbf{E}' = -\frac{1}{b}\ \frac{\partial\mathbf{B}'}{\partial\tau}, \quad \nabla' \mathrm{x} \mathbf{B}' = \frac{1}{b'}\left[\frac{\partial\mathbf{E}'}{\partial\tau} + 4\pi\rho'\mathbf{u}'\right]. \end{aligned} \tag{23}$$

It follows that Maxwell's equations are invariant under the transformations (15). *We see that the velocity of electromagnetic waves with respect to τ depends on the motion of the source, and* *their magnitude is always larger than c.* This observation may seem strange and even contradictory to the second postulate, but it is not. On closer inspection, we realize that the second postulate refers to the observer's point of view using measuring rods and clocks. Thus, there is no contradiction.

In the Michelson-Morley experiment, the source is at rest in the frame of the observer so that $u = 0$ and $b = c$. It follows that our approach explains the Michelson-Morley null result. It also provides agreement with the conceptual (but not technical) framework proposed by Ritz [10], namely, that the speed of light does not depend on the (proper) motion of the source. In this sense, both Einstein and Ritz are correct.

At the time of Einstein there was no reason to believe that the system would not necessarily unfold as the observer sees it. *The real physical question that arises is: Which velocity is most useful in understanding physical systems?* This is not an easy question since every known experiment which confirms the standard implementation can also be used to confirm the present approach. For example, the relativistic momentum increase is attributed to relativistic mass increase so that

$$\mathbf{p} = m\mathbf{w}, \qquad m = m_0\,[1 - w^2/c^2]^{-1/2}. \tag{24}$$

In our interpretation, we get

$$\mathbf{p} = m_0\mathbf{u}, \qquad \mathbf{u} = d\mathbf{x}/d\tau = \mathbf{w}\,[1 - w^2/c^2]^{-1/2}, \tag{25}$$

so there is no mass increase, the (proper) velocity increases. This implies that in particle lifetime measurements, the particle will have a fixed mass and fixed decay constant, independent of it's velocity. On the other hand, the particle can have speeds larger than the speed of light since it's velocity will be $d\mathbf{x}/d\tau$. In all cases where a length contraction or a time dilation is required in the standard approach, our approach leads to a statement about $\mathbf{u}$.

Note that, from $[1 + u^2/c^2]^{1/2} = [1 - w^2/c^2]^{-1/2}$, we have

$$[1 + u^2/c^2]^{1/2} = 1 + \frac{u^2}{c^2} - \frac{1}{8}\,\frac{u^4}{c^4} + \cdots, \tag{26a}$$

$$[1 - w^2/c^2]^{-1/2} = 1 + \frac{w^2}{c^2} + \frac{3}{8}\,\frac{w^4}{c^4} + \cdots. \tag{26b}$$

Thus, these two expressions agree in the low-velocity region. to make things even more difficult, it follows from $w = u[1 + u^2/c^2]^{-1/2}$ that

$$\begin{gathered} w/c = u/b, \\ \left[1 - w^2/c^2\right]^{-1/2} = [1 - u^2/b^2]^{-1/2}, \end{gathered} \tag{27}$$

so that all the results derived form the standard implementation of special relativity using w/c can also be consistently derived using u/b.

7. PROPER-TIME WAVE EQUATION

Returning to (21), we perform the standard manipulations, using

$$\mathbf{E} = -\frac{1}{b}\,\frac{\partial \mathbf{A}}{\partial \tau} - \nabla\Phi, \quad \mathbf{B} = \nabla \times \mathbf{A} \tag{28}$$

to obtain

$$\nabla\left[\nabla \cdot \mathbf{A} + \frac{1}{b}\,\frac{\partial \Phi}{\partial \tau}\right] = \frac{1}{b}\,\frac{\partial}{\partial \tau}\left[\frac{1}{b}\,\frac{\partial \mathbf{A}}{\partial \tau}\right] - \nabla^2 \mathbf{A} = \frac{1}{b}(4\pi \mathbf{J}), \tag{29}$$

and

$$-\nabla^2\Phi - \frac{1}{b}\,\frac{\partial}{\partial \tau}\,[\nabla \cdot \mathbf{A}] = 4\pi\rho. \tag{30}$$

Imposing the (proper-time) Lorentz gauge

$$\nabla \cdot \mathbf{A} + \frac{1}{b}\,\frac{\partial \Phi}{\partial \tau} = 0, \tag{31}$$

we get the wave equations

$$\frac{1}{b}\,\frac{\partial}{\partial \tau}\left[\frac{1}{b}\,\frac{\mathbf{A}}{\partial \tau}\right] - \nabla^2 \mathbf{A} = \frac{1}{b}(4\pi \mathbf{J}), \tag{32a}$$

$$\frac{1}{b}\,\frac{\partial}{\partial \tau}\left[\frac{1}{b}\,\frac{\partial \Phi}{\partial \tau}\right] - \nabla^2 \Phi = 4\pi\rho. \tag{32b}$$

Straightforward calculations lead to an equation of the form

$$-\nabla^2\Phi - \frac{1}{b^4}\left[\mathbf{u} \cdot \frac{d\mathbf{u}}{d\tau}\right]\frac{\partial \Phi}{\partial \tau} + \frac{1}{b^2}\,\frac{\partial^2 \Phi}{\partial \tau^2} = 4\pi\rho. \tag{33}$$

We get a similar equation for $\mathbf{A}$.

The dissipative term is zero if $\mathbf{u}$ is constant and arises instantaneously with acceleration. This is what we expect of a radiation reaction term (see Wheeler and Feynman [11]). It is also of interest to observe that we have made no assumptions about the structure of the source.

In order to solve (33), the simplest assumption is that $a^2 = -1/b^2[\mathbf{u}\cdot d\mathbf{u}/d\tau]$ and $1/b^2$ may be treated as constants. We can then solve the equation

$$-\nabla^2 G - \frac{a^2}{b^2}\,\frac{\partial G}{\partial \tau} + \frac{1}{b^2}\,\frac{\partial^2 G}{\partial \tau^2} = 4\pi\delta(\mathbf{r}-\mathbf{r}_\circ)\delta(\tau-\tau_\circ). \tag{34}$$

If $R = |\mathbf{r}-\mathbf{r}_\circ|$ and $t = \tau - \tau_0$, the standard solution ([12], p. 868) is

$$G(R,t) = G_1(R,t) + G_2(R,t),$$

where, setting $h(R,t) = (t^2 - R^2/b^2)^{1/2}$,

$$G_1(R,t) = 1/R \exp\left(-\frac{1}{2}\,a^2 t\right)\delta(t - R/b), \tag{35a}$$

$$G_2(R,t) = -a^2[2bh(R,t)]^{-1}\exp\{-\frac{1}{2}\,a^2 t\} I_1\left(\frac{1}{2}\,a^2 h(R,t)\right) U(bt-R), \tag{35b}$$

with

$$\mathbf{U}(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0, \end{cases} \tag{35c}$$

representing the step function, and I_1 is a modified Bessel function, $I_1(x) = -iJ_1(ix)$. We have taken the retarded solution because it is associated with the particle (moving forward in proper time). Assuming $\rho(\mathbf{r}_\circ,\tau_\circ) = q\delta(\mathbf{r}_\circ)$, we find

$$\begin{aligned}\Phi(\mathbf{r},\tau) &\\ &= \frac{1}{4\pi}\int d\tau_o d\mathbf{r}_\circ\{G_1(|\mathbf{r}-\mathbf{r}_\circ|, \tau-\tau_0) + G_2(|\mathbf{r}-\mathbf{r}_\circ|, \tau-\tau_0)\}\rho(\mathbf{r}_\circ,\tau_0) \\ &= \Phi_1 + \Phi_2,\end{aligned} \tag{36}$$

where

$$\Phi_1(\mathbf{r},\tau) = q(1/r)\exp\{-(a^2/2b)r\}, \tag{37a}$$

$$\begin{aligned}\Phi_2(\mathbf{r},\tau) = \frac{qa^2}{2}\int_0^\tau d\tau_o\, h(r,t)^{-1} I_1\left[\frac{1}{2}\,a^2 h(r,t)\right] \\ \exp\left\{-\frac{1}{2}\,a^2(\tau-\tau_o)\right\}U(b(\tau-\tau_o)-r).\end{aligned} \tag{37b}$$

Note that, if we let $\rho(\mathbf{r}_\circ, \tau_\circ) = q\delta(\mathbf{r}_\circ - \mathbf{u}\tau_\circ)$, then $1/r$ becomes $1/s$, where $s = r - (\mathbf{r}.\mathbf{u})/b$ and the exponential term becomes a complicated function of r, $\tau_\circ, \tau$ and $\mathbf{u}$.

The exponential term in (37a) can be both positive and negative. On the average, its value will be zero and we reproduce the standard radiation rate. A better approach is to set $\Phi = (b/c)^{1/2}\Lambda$, so that we solve:

$$\frac{1}{b^2}\frac{\partial^2 g}{\partial \tau^2} - \nabla^2 g + \left[\frac{\ddot{b}}{2b^3} - \frac{5\dot{b}^2}{4b^4}\right] g = 4\pi\delta(\mathbf{r} - \mathbf{r}_\circ)\delta(\tau - \tau_o). \tag{38}$$

We now assume that $\left[\ddot{b}/2b^3 - 5\dot{b}^2/4b^4\right]$ and $1/b^2$ may be approximately treated as constants. This gives $g(R,t) = g_1(R,t) + g_2(R,t)$, where

$$g_1(R,t) = 1/R\delta(t - R/b), \tag{39a}$$

$$g_2(R,t) = -\mu[h(R,t)]^{-1}\, L_1[\mu b h(R,t)]U(bt - R), \tag{39b}$$

$$\mu^2 = \left[\frac{\ddot{b}}{2b^3} - \frac{5\dot{b}^2}{4b^4}\right], \quad L_1 = J_1 \text{ if } \mu^2 > 0, \quad \mathrm{L}_1 = \mathrm{I}_1 \text{ if } \mu^2 < 0. \tag{39c}$$

It is now clear that (39a) will reproduce the standard radiation rate.

Assuming that $\rho(\mathbf{r}_\circ, \tau_o) = q\delta(\mathbf{r}_\circ - \mathbf{u}\tau_o)$, we get

$$\begin{aligned}\Phi(\mathbf{r},\tau) &= \frac{1}{4\pi}\int d\tau_o d\mathbf{r}_\circ \left[\frac{b}{c}\right]^{1/2} g_1(|\mathbf{r} - \mathbf{r}_\circ|,\, \tau - \tau_0) \\ &\quad + g_2(|\mathbf{r} - \mathbf{r}_\circ|,\, \tau - \tau_0)\{\rho(\mathbf{r}_\circ, \tau_0)\} \\ &= \Phi_1 + \Phi_2,\end{aligned} \tag{40}$$

with

$$\begin{aligned}\Phi_1(\mathbf{r},\tau) &= q(b/c)^{1/2}(1/s), \quad s = r - (\mathbf{r}.\mathbf{u})/b, \\ \Phi_2(\mathbf{r},\tau) &= -\int_0^\tau d\tau_\circ \mu \left[\frac{b}{c}\right]^{1/2} h(\mathbf{r} - \mathbf{u}\tau_\circ, t)^{-1} \\ &\quad \cdot L_1[\mu b h(\mathbf{r} - \mathbf{u}\tau_\circ, t)]U(bt - |\mathbf{r} - \mathbf{u}\tau_\circ|),\end{aligned} \tag{41}$$

where as before, $t = \tau - \tau_0$. We can now compute $\mathbf{A}$ from

$$\mathbf{A} = (\mathbf{u}/b)\,[\Phi_1(\mathbf{r},\tau) + \Phi_2(\mathbf{r},\tau)]. \tag{42}$$

8. DISCUSSION

To see our formulation in a different light, note that the Minkowski approach gives

$$(d\tau)^2 = (dt)^2 - 1/c^2(d\mathbf{x})^2 = (dt)^2\left(1 - \mathbf{w}^2/c^2\right), \quad \mathbf{w} = \frac{d\mathbf{x}}{dt}. \tag{43a}$$

We can also write this as

$$(dt)^2 = (d\tau)^2 + 1/c^2(d\mathbf{x})^2 = (d\tau)^2\left(1 + \mathbf{u}^2/c^2\right), \quad \mathbf{u} = \frac{d\mathbf{x}}{d\tau}. \tag{43b}$$

It is now clear that the special theory of relativity can also be formulated in Euclidean space provided we standardize time and velocity using the proper time and proper velocity of the source for all observers.

As is to be expected, our group (Eq. 15) is very close to the Lorentz group in that it is equivalent (but not identical) if the source has constant velocity. This is easy to see since then, the source is just another inertial frame. Note that there is no "geometrical time" in our formulation, τ *replaces* t *as one* of the physical observables defining the state of the system, so that even when the source has constant velocity, the physical interpretation is different.

When the source is accelerating, the group generators are nonlocal. This means that the transformation theory is quite distinct from that of the theories based on the use of historical time. We will discuss the details of the group and it's Lie algebra in a separate publication.

The fact that $w/c = u/b$ and $1/c\partial_t = 1/b\partial_\tau$, has some very interesting implications for all areas where classical electrodynamics is used. In each case, using the above two facts, we can construct a dual theory that is mathematically equivalent but which requires a completely different physical interpretation. For example, all the formula related to the Doppler effect (giving the apparent frequency of a moving light source) may be formulated as above and will give the same numerical values. This creates an interesting dilemma for cosmologists, since in the dual interpretation, the source can move faster than the speed of light compared to the observer. Let us consider another example which goes to the heart of the issue. In the standard approach, the Lorentz force can be written as

$$\begin{aligned} m\frac{d\mathbf{u}}{dt} &= \mathbf{E} + \mathbf{w}\times\mathbf{B}, \quad \mathbf{u} = \frac{d\mathbf{x}}{d\tau}, \quad \mathbf{w} = \frac{d\mathbf{x}}{dt}, \\ \mathbf{E} &= -\frac{1}{c}\,\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi. \end{aligned} \tag{44a}$$

Using $w/c = u/b$, $1/c\partial_t = 1/b\partial_\tau$, it is easy to see that this same equation can be written as

$$m\frac{c}{b}\frac{d\mathbf{u}}{d\tau} = \mathbf{E} + \mathbf{u}\times\mathbf{B}, \quad \mathbf{E} = -\frac{1}{b}\frac{\partial \mathbf{A}}{\partial \tau} - \nabla\Phi. \tag{44b}$$

These two equations are mathematically identical however, they are physically different and have different interpretations. The second equation is nonlinear in $\mathbf{u}$ (because of the b terms). It represents the force a local observer would obtain using his phase space and the proper clock of the particle via a canonical transformation of variables. This second equation will be derived in a different way from a theory in which the region in the immediate neighborhood of the particle is distorted (curved) because of the interaction.

It is clear that, in general, equation (33) cannot be solved so that some simplifying assumptions are required. When $a^2 = 0$, we see from (35a) that our result will lead to the standard Liénard-Wiechert potentials and hence, produces the standard radiation rates. It is not difficult to show that under fairly general conditions, $|a^2|$ will always be small compared to unity and that a^2 can be positive or negative. It should also be noted that (35a) is Coulomb/Yukawa-like, with $a^2/2b$ acting as an *effective mass*. This term is a consequence of the fact that our wave equation now has both a wave and a diffusion component. The second term represents the wake associated with the point source acceleration. For $b(\tau - \tau_\circ) \gg r$, this term yields the diffusion approximation and looks like a Gaussian pulse.

In (39c) we can also write μ^2 as

$$\mu^2 = \left[\frac{(\dot{\mathbf{u}})^2}{2b^4} + \frac{\mathbf{u}\cdot\ddot{\mathbf{u}}}{4b^4} - \frac{5(\mathbf{u}\cdot\dot{\mathbf{u}})^2}{4b^6}\right], \tag{45}$$

and it is now clear that the second term is the only one that can change sign. The first is always positive while the last is always negative. In the synchrotron case, the last term is zero. When μ^2 is positive we have a Klein-Gordon type equation. This has some interesting possibilities for the study of the electromagnetic mass, electron self-energy and stability in the classical case. In particular, since we are only interested in retarded solutions and Maxwell's equations are first order (in time), we should be looking for a first order equation that gives the retarded solutions directly. In our case, this leads to

$$i\frac{\partial g}{\partial \tau} - b\sqrt{-\nabla^2 + \mu^2}\, g = 4\pi\delta(\mathbf{r} - \mathbf{r}_\circ)\delta(\tau - \tau_\circ). \tag{46}$$

To the writer's knowledge, there is no published analytical solution to this equation in the literature. Such a solution would also be of interest for the foundations of relativistic quantum mechanics.

9. CONCLUSION

We have constructed a direct implementation of the first two postulates of the special theory of relativity without assuming that time be treated as a fourth coordinate. We obtain a different transformation theory which fixes the proper time of the source for all observers. This leads to a generalization of Maxwell's equations and a new wave equation which depends on the motion of the source. It is shown that the speed of light with respect to proper time will depend on the motion of the source as suggested by Ritz and is not in contradiction with the second postulate. Our wave equation is solved using two different approximation assumptions. Each approach allows us to recover the standard theory, but also gives an additional term which is interpreted as a wake or shock wave caused by the particle acceleration. This is equivalent to a damping term which is of both the type and order of magnitude to directly account for radiation reaction without any assumption about the structure of the source.

Our approach is motivated by the observation (of Feynman) *that time is both a physical quantity and an index which identifies the order of physical events.* The successful construction of the Feynman time-ordered operator calculus [13], its intuitive physical content and our ability to use it to prove the Dyson conjecture [14], convinced us that the use of time as a fourth coordinate may well be a major cause of problems in the merging of relativity with quantum mechanics.

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