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QUANTUM-MECHANICAL UNCERTAINTY RELATIONS AS A CONSEQUENCE OF THE POSTULATES OF N.A.KOZYREV'S CAUSAL MECHANICS; FORCES IN CAUSAL MECHANICS

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This paper consists of four independent sections. In the first one Heisenberg's uncertainty relations are derived on the basis of the fundamental postulates of N.A.Kozyrev's causal mechanics. The second section contains a definition of the course of time c_2 differing from that introduced by N.A.Kozyrev. In the third section possible generalizations of Kozyrev's expressions for the additional forces acting in causal links in causal mechanics are proposed. The fourth section analyses the inaccuracy of force representation in classical mechanics related to the neglect of time intervals between causes and effects.

1. Causal mechanics and the quantum-mechanical uncertainty relations

N.A.Kozyrev's causal mechanics (Kozyrev 1991) begins with the postulates claiming that in an elementary cause-effect link the cause and effect points are separated by an arbitrarily small but nonzero space and time differences δx and δt whose ratio is a fundamental constant called the *course of time* c_2:

$$c_2 = \delta x / \delta t \equiv \text{const} . \tag{1.1}$$

The constant c_2 is assumed to be pseudoscalar. Its pseudoscalarity is related to the same property of the quantity δt. However, the statement that δt is pseudoscalar, is, in our view, not sufficiently justified. To "avade" the question of whether δt is a pseudoscalar or a true scalar, let us pass in law (1.1) to the magnitudes of the quantities:

$$|c_2| = |\delta x| / |\delta t| \equiv \text{const} . \tag{1.2}$$

The physical meaning of the quantities δx and δt is not described in detail in causal mechanics. We assign them the meaning allowing one to establish a relation between causal mechanics and quantum physics.

Let space and time form a unified four-dimensional manifold possessing the proper Euclidean geometry including both space and time variables (in what follows it does not matter which global geometry, proper Euclidean or pseudoeuclidean, is used, since the spatial and temporal quantities are considered separately in the present section).

We define a "*collision*" as an interaction of material points (particles) that they approach each other to the minimum possible spatial and temporal distances. It should be noted that the minimum distances between particles may be different in different "collision" acts, but they are undoubtedly nonzero since in a Euclidean continuum different points are always separated by a nonzero interval.

Assume that the space and time coordinates of "colliding" material points are independent random variables and that the quantities $|\delta x|$ and $|\delta t|$ are quantum-mechanical uncertainties (i.e., root-mean-square values) in space and time distances between the "collided" particles:

$$|\delta x| = \sqrt{\overline{(\vec{r}_1 - \vec{r}_2)^2}}; \quad |\delta t| = \sqrt{\overline{(t_1 - t_2)^2}}, \tag{1.3}$$

where $\vec{r}_1$, t_1, $\vec{r}_2$, t_2 are the spatial radius-vectors and time coordinates of the "collided" particles; the bars denote the procedure of averaging over all the possible values.

Assume that the random quantities $\vec{r}_1$ and $\vec{r}_2$, as well as t_1 and t_2, are characterized by the same distribution densities and average values. The space-time point coinciding with the average position of both particles, will be called the *collision point*. It is this point that in a macroscopic description is considered to be the place where the two particles "collide". The spatial radius-vector $\vec{r}$ and the time coordinate t of the collision point are

$$\vec{r} = \bar{\vec{r}}_1 = \bar{\vec{r}}_2; \quad t = \bar{t}_1 = \bar{t}_2. \tag{1.4}$$

The root-mean-square deviations from the collision point are equal for the two particles due to identity of their density distributions, and they, in both space and time directions, are, respectively,

$$\Delta r = \sqrt{\overline{(\vec{r}_1 - \vec{r})^2}} = \sqrt{\overline{(\vec{r}_2 - \vec{r})^2}};$$
$$\Delta t = \sqrt{\overline{(t_1 - t)^2}} = \sqrt{\overline{(t_2 - t)^2}}. \tag{1.5}$$

By (1.3) - (1.5) and due to independence of the random quantities $\vec{r}_1$ and $\vec{r}_2$ one can write:

$$\begin{aligned} |\delta x|^2 &= \overline{(\vec{r}_1 - \vec{r}_2)^2} = \overline{[(\vec{r}_1 - \vec{r}) - (\vec{r}_2 - \vec{r})]^2} \\ &= \overline{(\vec{r}_1 - \vec{r})^2} - 2\overline{(\vec{r}_1 - \vec{r}) \cdot (\vec{r}_2 - \vec{r})} + \overline{(\vec{r}_2 - \vec{r})^2} \\ &= 2(\Delta r)^2 - 2\overline{(\vec{r}_1 - \vec{r})} \cdot \overline{(\vec{r}_2 - \vec{r})} = 2(\Delta r)^2. \end{aligned}$$

Hence the particle spatial position uncertainty is connected with the quantity $|\delta x|$ by the relation

$$\Delta r = \frac{1}{\sqrt{2}}|\delta x|. \tag{1.6}$$

Similarly for the particle's temporal coordinate uncertainty one can obtain the following relation involving $|\delta t|$:

$$\Delta t = \frac{1}{\sqrt{2}}|\delta t|. \tag{1.7}$$

While describing a "collision" at the macroscopic level, a single point introduced above by (1.4) is assumed to be the force application point for both particles. Meanwhile, the real positions of particles in space and time and consequently their force application points may not coincide with the collision point. The inaccuracy of the force application points determination leads to the inaccuracies of particle energies and momenta. Besides, the energy determination error is equal to the work to be done by the force displacing a particle from the collision point to that of its real location. And the momentum determination error is equal to an additional momentum which the particle should have gained under the action of the above force for a time interval between the real interaction instant and that corresponding to the collision point. Thus, the inaccuracies of energy and momentum determination in a separate "collision" act are equal to $\vec{F}_1 \cdot (\vec{r}_1 - \vec{r})$ and $\vec{F}_1 (t_1 - t)$, respectively, for one particle, and $\vec{F}_2 \cdot (\vec{r}_2 - \vec{r})$ and $\vec{F}_2 (t_2 - t)$ for the other, where $\vec{F}_1$ and $\vec{F}_2$ are forces acting on the first and second particles. The root-mean-square values of these quantities may be identified with quantum-mechanical uncertainties in particle energies and momenta. Let us calculate them.

Assume that the particles interact by the forces described by Newton's classical mechanics, i.e. the forces which are equal in magnitude, oppositely directed and have a common line of action, namely, the straight line passing through both particles (the forces introduced in causal mechanics are neglected due to their smallness). Such forces may be represented in the form

$$\vec{F}_1 = \pm F \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}; \quad \vec{F}_2 = -\vec{F}_1, \tag{1.8}$$

where F is the magnitude of the forces $\vec{F}_1$ and $\vec{F}_2$ and $(\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|$ is the direction unit vector; the plus and minus signs correspond to the cases of particle repulsion and attraction, respectively.

While calculating the energy uncertainties, we restrict ourselves to the case when the particles in "collision" are situated on the same line with the collision point (whose position may be different for different "collisions"). Since in this case the forces $\vec{F}_1$ and $\vec{F}_2$ are oriented along the same line, the direction unit vector in (1.8) coincides up to a sign with the vectors $(\vec{r}_1 - \vec{r})/|\vec{r}_1 - \vec{r}|$ and $(\vec{r}_2 - \vec{r})/|\vec{r}_2 - \vec{r}|$, hence (1.8) may be rewritten in the form

$$\vec{F}_1 = \pm F \frac{\vec{r}_1 - \vec{r}}{|\vec{r}_1 - \vec{r}|} = \pm F \frac{\vec{r}_2 - \vec{r}}{|\vec{r}_2 - \vec{r}|}; \quad \vec{F}_2 = -\vec{F}_1 \tag{1.9}$$

(here and in Eq. (1.11) presented below the sign of $\vec{F}_1$ may differ from that in formula (1.8)). For such a representation of the forces $\vec{F}_1$ and $\vec{F}_2$ one easily calculates the energy value uncertainty ΔE, the same for both particles:

$$\Delta E = \sqrt{\overline{\left[\vec{F}_i \cdot (\vec{r}_i - \vec{r})\right]^2}} = \sqrt{\overline{\left[\pm F \frac{(\vec{r}_i - \vec{r}) \cdot (\vec{r}_i - \vec{r})}{|\vec{r}_i - \vec{r}|}\right]^2}}$$

$$= F^* \sqrt{\overline{(\vec{r}_i - \vec{r})^2}} = F^* \Delta r, \qquad (1.10)$$

where F* is the value of F at a certain mean point; i = 1, 2; here the mean-value theorem and the first formula from (1.5) are used.

Now let us calculate the momentum uncertainty. Aiming to compare the result to be obtained with the corresponding result of quantum mechanics, we perform calculations in the one-dimensional case, as was done in the book by Landau and Lifshitz (1989). Let the "colliding" particles and the collision point be situated on a single line parallel to the z coordinate axis. Then the forces $\vec{F}_1$ and $\vec{F}_2$ described by (1.8) can be represented in the form

$$\vec{F}_1 = \pm F\vec{k}; \quad \vec{F}_2 = -\vec{F}_1, \qquad (1.11)$$

where $\vec{k}$ is the direction unit vector of the z axis. In this case the uncertainty Δp_z of the momentum z component, having the same value for both "colliding" particles, is

$$\Delta p_z = \vec{k} \cdot \sqrt{\overline{\left[\vec{F}_i (t_i - t)\right]^2}} = \sqrt{\overline{\left[\pm F\vec{k} \cdot \vec{k}(t_i - t)\right]^2}}$$

$$= F^{**} \sqrt{\overline{(t_i - t)^2}} = F^{**} \Delta t, \qquad (1.12)$$

where F** is the value of F at a certain mean point; i = 1, 2; the mean-value theorem and the second formula from (1.5) are used. In this case the z coordinate uncertainty Δz, having the same value for both colliding particles, is

$$\Delta z = \sqrt{\overline{(z_1 - z)^2}} = \sqrt{\overline{(z_2 - z)^2}} = \Delta r, \qquad (1.13)$$

where z_1, z_2, z are the z coordinates of the particles in "collision" and that of the collision point, respectively.

Let us specify the values of the forces $\vec{F}_1$ and $\vec{F}_2$. We shall assume that the particles bear electric charges e or –e (–e being the electron charge), interact only by electric forces and, while "colliding", are mutually at rest. In this case their interaction is performed by the Coulomb forces described by the expression (1.8), with the magnitude

$$F = \frac{e^2}{4\pi\varepsilon_0 |\vec{r}_1 - \vec{r}_2|^2},$$

where ε_0 is the vacuum permittivity. In what follows we use only that magnitude value of forces which corresponds to the particle spacing $|\vec{r}_1 - \vec{r}_2|$ equal to $|\delta x|$. It is this force magnitude value that is further designated as F:

$$F = \frac{e^2}{4\pi\varepsilon_0|\delta x|^2} \ . \qquad (1.14)$$

Now let us form the product of the force magnitude F and the uncertainties in the space and time coordinates of the particles. Taking into account the dependences (1.6), (1.7), (1.14) and the definition of the course of time c_2, we obtain

$$F\,\Delta r\,\Delta t = \frac{1}{2}F|\delta x||\delta t| = \frac{e^2|\delta t|}{8\pi\varepsilon_0|\delta x|} = \frac{e^2}{8\pi\varepsilon_0|c_2|} = \alpha\frac{\hbar}{2}\frac{c}{|c_2|}\,, \qquad (1.15)$$

where $\alpha = e^2/(4\pi\varepsilon_0\hbar c) \approx 1/137$ is the fine structure constant; $\hbar = h/(2\pi)$ is the Planck constant and c is the velocity of light in vacuum.

It is evident that the parameters F* and F** involved in Eqs. (1.10) and (1.12), can be set equal to F. Hence from (1.10), (1.12), (1.13), (1.15) it follows that

$$\Delta E\,\Delta t = \alpha\frac{\hbar}{2}\frac{c}{|c_2|}; \quad \Delta p_z\,\Delta z = \alpha\frac{\hbar}{2}\frac{c}{|c_2|}. \qquad (1.16)$$

One of the uncertainty relations of quantum mechanics, written for the minimum possible values of the uncertainties, is of the form

$$\Delta p_z \Delta z = \frac{\hbar}{2}. \qquad (1.17)$$

Comparing the second relation of (1.16) with (1.17), we find:

$$\frac{|c_2|}{c} = \alpha \approx \frac{1}{137}; \quad |c_2| = \alpha c \approx 2187.7\,\text{km/s}. \qquad (1.18)$$

The fact that the constant c_2, the fundamental quantitative characteristic of causal mechanics, is represented in the form of a product of fundamental constants, confirms the validity of one of the starting points of Kozyrev's theory, namely, that this constant is fundamental.

The above numerical value of the constant c_2 is in agreement with the value obtained by N.A.Kozyrev experimentally by measuring additional forces in mechanical systems (Kozyrev 1991, pp. 367, 382). The fact that the experimental value of c proved to be precisely the one, allowed him to adopt the relationship $|c_2| = \alpha c$ as an empirical fact.

The result expressed by (1.18) allows some points of quantum mechanics to be reviewed. The origin of the fine structure (dimensionless, fundamental) constant has been troubling physicists for long. Thus, R.Feynman (1985) named the question of how this number appears, one of the *greatest* damned mysteries of physics: a *magic number* which is given to us and which man does not understand at all. Relations (1.18) lift the veil of mystery around this number. According to N.A.Kozyrev, "...the presence of the dimensionless constant α ceases to be mysterious and becomes natural as a ratio of two fundamental velocities" (Kozyrev 1991, p. 367).

Equations (1.18) enable one to refine and reinterpret the uncertainty relation for energy and time. This relation, as applied to the minimum possible values of the uncertainties, is conventionally written in the form

$$\Delta E\,\Delta t \sim \hbar. \tag{1.19}$$

This relation, unlike (1.17), does not set an exact lower bound of the product of uncertainties but only its order of magnitude. The very quantities entering into (1.19) are treated differently from those appearing in (1.17). This is related to the fact that in quantum mechanics time is considered to be a determinate but not random variable. Hence the quantities ΔE and Δt are not understood conventionally, i.e., they are not regarded as root-mean-square deviations but, instead, as an energy measurement error and a duration of its measuring respectively (De Broglie 1982, Demutsky and Polovin 1992). It is easy to see that the difference in interpretations of quantum mechanical dependences (1.17) and (1.19) contradicts the relativistic symmetry of space and time. Equations (1.18) allow this contradiction to be eliminated. They and the first equality from (1.16) lead to the uncertainty relation for energy and time in the "standard" form relating to one another the minimum possible values of the root-mean-square deviations of the corresponding variables:

$$\Delta E\,\Delta t = \frac{\hbar}{2}. \tag{1.20}$$

Equations (1.18) and (1.15) yield one more uncertainty relation:

$$F\,\Delta r\,\Delta t = \frac{1}{2}F|\delta x||\delta t| = \frac{\hbar}{2}, \tag{1.21}$$

where a value with the dimension of action stands at the left-hand side.

Restrictions on the possible values of the quantities Δr and Δt can be obtained provided that the energy uncertainty does not exceed the rest energy of an electron:

$$\Delta E \le m_e c^2, \tag{1.22}$$

where m_e is the electron mass. This condition and Eqs.(1.6), (1.7), (1.14), (1.20), (1.21) lead to the following inequalities:

$$\begin{aligned} \Delta r &= \frac{1}{\sqrt{2}}|\delta x| \ge \frac{\alpha\hbar}{2m_e c} \approx 1.41\cdot 10^{-15}\,\text{m}; \\ \Delta t &= \frac{1}{\sqrt{2}}|\delta t| \ge \frac{\hbar}{2m_e c^2} \approx 6.44\cdot 10^{-22}\,\text{s}, \end{aligned} \tag{1.23}$$

where the quantity on the right side of the first inequality is equal to half the so-called classical radius of an electron.

The present section departs from the division of interacting material points into a cause and an effect, being of importance in causal mechanics (as the effect always comes after the cause). The interacting particles are equivalent in the above considerations and cannot be consistently divided into a cause and an effect, e.g., their time coordinates in "collision" equally probably satisfy both inequalities $t_1 > t_2$ and $t_2 > t_1$.

Making use of the uncertainty relation (1.17), we have proved the validity of Kozyrev's law (1.2) and confirmed that the course of time c_2 has just the value which N.A.Kozyrev ascribed to it on the basis of the results of macroscopic experiments. If the law (1.2), involving the constant c_2 given by (1.18), were assumed to be a fundamental postulate, the uncertainty relations (1.17), (1.20), (1.21) might be easily obtained. This means, in particular, that the quantum-mechanical uncertainty relations may be regarded as a consequence of the postulates of causal mechanics.

From the content of the present section it can be concluded that Kozyrev's causal mechanics is in agreement with quantum physics. Moreover, causal mechanics results in a new interpretation of Heisenberg's uncertainty relations. The latter may be treated as a consequence of the uncertainty in the space-time intervals in particle "collisions". The uncertainties obey the law (1.2) with the constant c_2 equal in magnitude to αc. This interpretation may obviously make us revise our attitude to the other conceptual statements of quantum mechanics as well.

2. On the time characteristic c_2 in N.A.Kozyrev's theory

An experiment for measuring the *course of time* c_2 was carried out by N.A.Kozyrev by weighing a rotating gyroscope with a vertically oriented axis (Kozyrev 1991). When vertical vibrations were introduced into the balance-gyroscope system, a change of the gyroscope weight was observed by the value of $\Delta\Phi$ proportional to its weight Φ and the linear rotation speed v of the rotor; the value of the parameter c_2 was calculated by the formula

$$\left|\Delta\Phi\right| = \frac{\pi}{c_2} v\,\Phi \tag{2.1}$$

and turned out to be about 2200km/s (Kozyrev 1991, pp.366-367, 382). N.A.Kozyrev treated this fact as appearance of additional forces neglected in classical mechanics. He postulated c_2 to be a pseudoscalar, since the effect changed sign when the physical system under investigation was replaced by a mirror-symmetric one.

The course of time c_2 is defined in causal mechanics as the rate of causal action realized in an elementary cause-and-effect link comprising two material points, those of the cause and the effect:

$$c_2 = \delta x / \delta t\,, \tag{2.2}$$

where δx and δt are arbitrarily small but nonzero space and time differences between the cause and effect points.

This definition assigns a clear physical meaning to the most important characteristic of time in causal mechanics. The validity of just this definition is supported by the results of the previous section where the quantity c_2 was proved to be a fundamental constant. Nevertheless, the above definition has a number of shortcomings.

1. The course of time c_2 is determined by Eq. (2.2) in terms of the quantities δx and δt eluding a direct experimental measurement.

2. Equation (2.2) is inconsistent with a pseudoscalar nature of c_2 (N.A.Kozyrev's assumption that the time interval δt is a pseudoscalar, is not sufficiently justified in his papers (Kozyrev 1991) and therefore cannot be taken for granted).

3. The definition under consideration leads to an inconsistency between the instantaneous character of action transmission via time through cosmic distances (Kozyrev and Nasonov 1978, 1980) and the finiteness of the action transmission velocity in an elementary cause-and-effect link.

4. Kozyrev (1991) has not presented a strictly logical transition from the definition (2.2) to the additional force formula (2.1) (such a transition is most likely impossible in principle, since with only a single scalar quantity (c_2) available no unambiguous conclusion concerning a vector quantity, i.e., the additional force, can be made). Hence the quantity c_2 appearing in (2.1) must not necessarily coincide with that defined by (2.2).

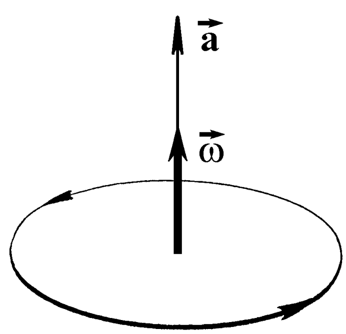

Fig.1. A pair of collinear vector $\vec{a}$ and pseudovector $\vec{\omega}$: The shown direction of the pseudovector $\vec{\omega}$ corresponds to the marked circle round travel direction in a right coordinate system.

In connection with the shortcomings of this definition it would be reasonable to try to formulate another definition of the course of time, retaining the essential features of the quantity c_2 described by Kozyrev (1991) but free of these shortcomings. Such a definition is suggested below.

Based on the propositions of causal mechanics, we shall assume that time interacts in different ways with right- and left-handed physical systems by its active properties. A pair $(\vec{a}, \vec{\omega})$ consisting of a vector $\vec{a}$ and a pseudovector $\vec{\omega}$ collinear to each other, is one of the simplest mathematical objects distinguishing the right from the left (Fig.1). (A simple example: a motion in the direction pointed by the vector $\vec{a}$ combined with a rotation defined by the pseudovector $\vec{\omega}$ is right-hand-screw if the directions of $\vec{a}$ and $\vec{\omega}$ coincide and left-hand-screw otherwise.) Assume that the c o u r s e o f t i m e is described exactly by such a mathematical object. Then it may obviously manifest itself in physical systems whose kinematics is characterized by a similar vector pair. This is just the case in the experiment with a vibrating gyroscope described above, where such a kinematic pair is formed by the gyroscope acceleration $\vec{a} = a\vec{k}$ due to its vibration and the angular velocity of its rotation $\vec{\omega} = \omega\vec{k}$ (here a is a scalar, ω is a pseudoscalar, $\vec{k}$ is the direction unit vector of the rotation axis).

It can be assumed that the action of the physical properties of time on the gyroscope results in appearance of the addition Δa and $\Delta\omega$ to the values of a and ω, which are monotonic functions of these values, satisfy the condition $\Delta a = \Delta\omega = 0$ if $a\omega = 0$ and have signs depending on the mutual orientation of the vectors $\vec{a}$ and $\vec{\omega}$. Then we can write down in the linear approximation in a and ω:

$$\Delta a = \pm k_a a\omega; \quad \Delta\omega = \pm k_\omega a\omega, \qquad (2.3)$$

where k_a and k_ω are dimensional coefficients; the signs are positive for one mutual orientation of the vectors $\vec{a}$ and $\vec{\omega}$ and negative for the other.

In gyroscope vibration its acceleration $\vec{a}$ regularly changes its sign, whereas the angular velocity $\vec{\omega}$ remains unchanged. The time average of the addition Δa turns out to be nonzero despite the fact that the average acceleration being zero. This is related to the fact that the sign of Δa is the same for any half-period of vibration because it depends on both the sign of a and the mutual orientation of $\vec{a}$ and $\vec{\omega}$ changing together with the sign changing of a. Multiplying the mean value of Δa by the gyroscope rotor mass, we obtain the mean value of the additional force acting on the gyroscope:

$$|\Delta\Phi| = \frac{k_a \overline{|a|}}{Rg} v\Phi \ . \qquad (2.4)$$

Here the relation $\omega = v/R$ is used; besides, the rotor mass is taken to be equal to the whole gyroscope mass Φ/g as it was done by Kozyrev (1991); R and v are the mean values of the rotor radius and its linear rotation velocity, respectively; Φ is the gyroscope weight; g is the free fall acceleration; an overbar denotes the time averaging operation. We do not specify the sign of $\Delta\Phi$, since the observable may always be fited by choosing the required sign in (2.3). The quantity $\Delta\Phi$ may be obviously interpreted as a change of the gyroscope weight.

Let us compare (2.4) with the relation (2.1) obtained experimentally. It is seen that Eq.(2.4) incorporates the same dependence of the additional force on the linear rotation velocity of the rotor v and the gyroscope weight Φ as does the relation (2.1). This suggests that the first equality from (2.3) should be valid, since it is just the basis for Eq.(2.4). It should be emphasized that a distinction between the factors by $v\Phi$ in Eqs. (2.1) and (2.4) does not argue against this conclusion. The point is that the relation (2.1), being just an expression of particular experimental data, is of restricted nature. In particular, it neglects a dependence of the additional force on vibration intensity and on the geometric parameters of the gyroscope, which should occur in reality and is apparently taken into account by just the above factor in Eq.(2.4).

Thus, we have confirmed the validity of the first equality from (2.3). It is clear that the coefficient k_a appearing in this equality may depend on the vibration characteristics and the gyroscope size. Assume that the second equality in (2.3) holds as well and the coefficient k_ω in it depends on the system properties in the same way as the coefficient k_a ($\Delta\omega$ was not measured by Kozyrev, hence this assumption cannot be compared with the experimental data). Then the ratio $\Delta a/\Delta\omega$ is a pseudoscalar having the dimension of velocity and independent of the specific properties of the system under study.

It is natural to adopt the quantity $\Delta a/\Delta\omega$ to be the course of time c_2. One easily assures that it is free of the mentioned shortcomings of the "old" definition based on the relation (2.2).

The proposed approach to defining the course of time admits extension to physical systems unrelated to rotating bodies. Other quantities, e.g., energy flux density and the density of volume force moments, can play for such systems the same role as the pair $(\vec{a}, \vec{\omega})$.

Remark. The content of the present section follows a manuscript of April 1979. The manuscript was discussed with N.A.Kozyrev who made the following two remarks.

1. In the case depicted in Fig. 1 the momentum conservation law appears to be violated due to an uncompensated force acting on the system if $\vec{a}$ is an acceleration. Meanwhile, the validity of this law has been verified to a high accuracy in special experiments when both the source of vibration and the gyroscope were placed on the same balance pan. In such experiments additional forces were not detected.

2. Equation (2.4) contains the rotor radius R. To bring it to the form (2.1), it is necessary to assume that $k_a \sim R$. However, in such a case the physical meaning of formula (2.3) is unclear. Experiments with gyroscopes whose rotor had the shape of a thin-walled glass (so that the condition $R \equiv const$ was fulfilled to a good accuracy), as well as an analysis of planet figure asymmetries and an investigation of the latitudinal dependence of the gyroscope weight change effect convince that the ratio $v/R \equiv \omega$ in formula (2.4) should be replaced by the linear velocity v of the points of the rotor.

Figure 2, depicting a possible system of vectors for the cause-and-effect link as a whole, answers N.A.Kozyrev's first remark. It is seen that the uncompensated forces are absent in such a system and the momentum conservation law remains valid. However, the author has no answer to the second remark.

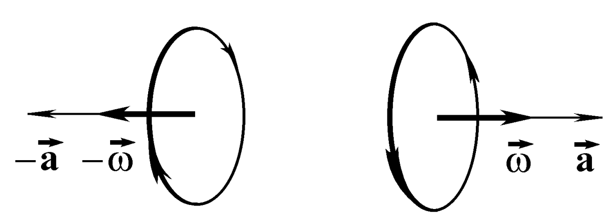

Fig.2. A possible system of vectors for two interacting objects.

3. Forces due to the action of time

According to N.A.Kozyrev's causal mechanics (Kozyrev 1991), the action of time on our World is realized in cause-and-effect relations. Due to this action in the causal relations there appear small forces in addition to the conventional ones taken into account by classical mechanics. These forces are directed in such a way that they should lead to a mirror asymmetry between the cause and the effect, responsible for an objective difference between them in causal mechanics.

N.A.Kozyrev's papers specify the values of the additional forces as applied to the case when a rotating perfect top is incorporated in the cause-and-effect link as one of the components. We would like to suggest possible generalizations to the cases of arbitrary pairs of interacting material points.

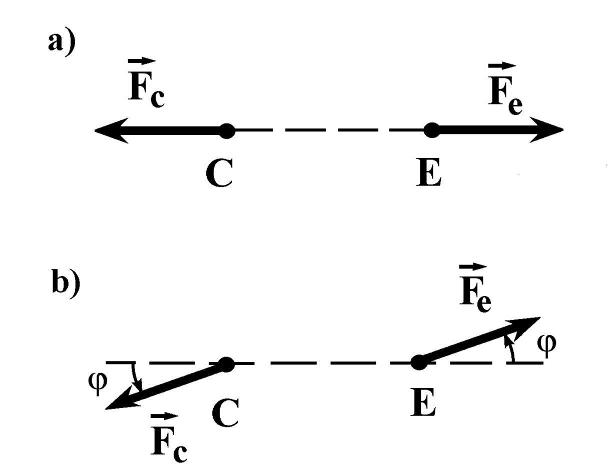

Fig.3. An elementary cause-and-effect link consisting of two material points:
a — the forces $\vec{F}_c$ and $\vec{F}_e$ having a common line of action; b — the lines of action of the forces $\vec{F}_e$ and $\vec{F}_c$ are parallel but do not coincide; C — the cause, E — the effect; $\vec{F}_c$ and $\vec{F}_e$ — the action and reaction forces; $\vec{F}_c = -\vec{F}_e$; φ — the forces' deflection angle from the straight line CE.

Following N.A.Kozyrev, let us consider an elementary cause-and-effect link consisting of two material points, a cause point and an effect point, with no other material body between them. We shall assume that the cause point C acts on the effect point E by the force $\vec{F}_e$, and the effect E reacts on the cause C by the reaction force $\vec{F}_c$. According to Newton's third law the forces of action and reaction are equal in magnitude and opposite in direction, i.e., $\vec{F}_c = -\vec{F}_e$. In addition to Newton's third law, theoretical mechanics always assumes that *the interaction forces between any two internal points of the system act along one and the same line* (Polyakhov et al. 1985, p.137). As applied to the cause-and-effect link under consideration, this assumption means that the forces $\vec{F}_c$ and $\vec{F}_e$ are directed along the straight line connecting the points C and E (Fig.3a).

Let us note the fact that classical mechanics does not consider the assumption of orientation of internal forces to be such a fundamental law of nature as Newton's laws. Moreover, theories lacking such assumptions have been constructed for long in continuum mechanics, one of the branches of classical mechanics (Sedov 1983). The moment theory of elasticity, elaborated as early as at the dawn of the twentieth century, provides an example for such a theory (Nowacki 1970, Chapter 13). On breaking with that assumption, the forces of action and reaction may appear to be directed along collinear but uncoinciding lines (Fig.3b). Newton's third law remains valid, as before, i.e., $\vec{F}_c = -\vec{F}_e$.

Suppose that the "interference" of time in the causal relation leads just to breaking the above assumption. Namely, let us assume that the action of time manifests itself in a deviation of the vectors of the forces $\vec{F}_c$ and $\vec{F}_e$ from the straight line by the same angle $\varphi \in [0, \pi/2]$ to opposite sides. Three possible versions of such a deviation can be suggested.

Version 1. Let the deviations of the forces $\vec{F}_c$ and $\vec{F}_e$ from the straight line CE be accompanied by their rotation about this line in the same direction with a certain angular velocity $\vec{\omega}$ (Fig.4a). In this case the two components of the cause-and-effect link turn out to be objectively different. Indeed, looking at one of the components from the place where the other is located, we see the rotation of the force vector occurring anticlockwise, and looking at the other component from where the first component is located, we see rotation of the force vector occurring clockwise. So this version relates the difference between the cause and the effect to that between the right and the left in our World, as it should be the case in accord with the fundamentals of Kozyrev's causal mechanics.

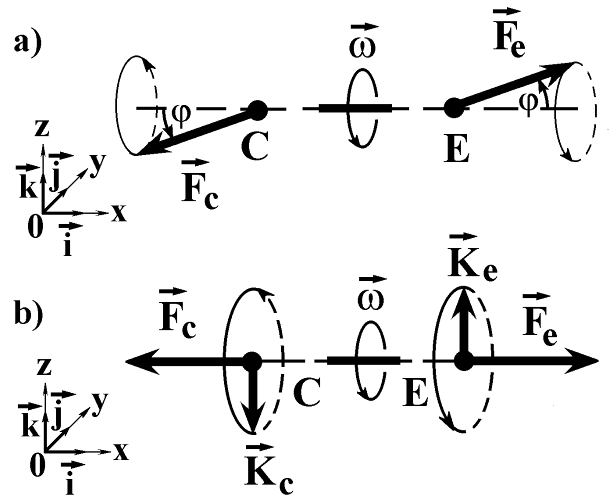

Fig.4. A possible influence of time on a causal link: a — the deflection of the forces $\vec{F}_c$ and $\vec{F}_e$ from the straight line CE by the angle $\varphi \in [0, \pi/2]$ with their rotation around this line with the angular velocity $\vec{\omega}$ ($\vec{F}_c = -\vec{F}_e$); b — appearance of the additional forces $\vec{K}_c$ and $\vec{K}_e$ perpendicular to the line CE and rotating around it with the angular velocity $\vec{\omega}$ ($\vec{K}_e = -\vec{K}_e$); this case coincides with "a" in the linear approximation in φ for small φ and $|\vec{K}_c| = |\vec{F}_c| \tan\varphi$, $|\vec{K}_e| =$

Let us introduce a Cartesian right-handed rectangular coordinate system {O, x, y, z} with the Ox axis parallel to the straight line CE pointed from the cause to the effect, as shown in Fig.4a. Denote the unit vectors of the Ox, Oy, Oz coordinate axes by $\vec{i}, \vec{j}, \vec{k}$, respectively. Then the force $\vec{F}_e$ in a position deflected from the line CE can be represented in the form of the sum of three components along the coordinate axes:

$$\vec{F}_e = F_{ex}\vec{i} + F_{ey}\vec{j} + F_{ez}\vec{k}, \tag{3.1}$$

its projections on the coordinate axes being described by the formula

$$\begin{aligned} F_{ex} &= F\cos\theta; \\ F_{ey} &= F\sin\theta\cos[\omega_x(t - t_0)]; \\ F_{ez} &= F\sin\theta\sin[\omega_x(t - t_0)] \ . \end{aligned} \tag{3.2}$$

Here $F = |\vec{F}_e|$ is the force magnitude; the force magnitude in this case coincides with that given by classical mechanics; θ is the angle between the unit vector $\vec{i}$ and the force vector $\vec{F}_e$, $0 \le \theta \le \pi$ ($\theta = \varphi$ for $0 \le \theta \le \pi/2$, which occurs when the effect repels the cause, and $\theta = \pi - \varphi$ for $\pi/2 \le \theta \le \pi$, which corresponds to attraction of the effect to the cause, where φ is the deflection angle of the forces $\vec{F}_e$ and $\vec{F}_c$ from the line CE); $\omega_x = \vec{\omega} \cdot \vec{i}$ is the projection of the angular velocity pseudovector $\vec{\omega}$ on the Ox axis (in our case $\vec{\omega} = \omega_x \vec{i}$); t_0 is a time parameter characterizing the rotation phase of the force $\vec{F}_e$. Certainly the force $\vec{F}_c$ may be decomposed into similar components differing from those of $\vec{F}_e$ only by sign.

For a small angle φ ($\varphi << 1$) this version of the action of time can be presented (in the linear approximation in φ) as the appearance of small, oppositely directed additional forces $\vec{K}_c$ and $\vec{K}_e$ applied to the cause C and the effect E. We denote these forces by the letter K after Kozyrev's name. The forces $\vec{K}_c$ and $\vec{K}_e$ are orthogonal to the straight line CE, rotate about it with the angular velocity $\vec{\omega}$ and satisfy the relations

$$\left|\vec{K}_c\right| = \left|\vec{F}_c\right| \tan\varphi; \quad \left|\vec{K}_e\right| = \left|\vec{F}_e\right| \tan\varphi, \tag{3.3}$$

where the forces $\vec{F}_c$ and $\vec{F}_e$ are now directed along the straight line CE (Fig.4b). Here

$$\left|\vec{K}_c\right| << \left|\vec{F}_c\right|, \quad \left|\vec{K}_e\right| << \left|\vec{F}_e\right|, \quad \vec{K}_c = -\vec{K}_e.$$

As seen from (3.2), in the version under consideration the three scalar quantities: the angle φ (or θ), the angular velocity projection ω_x and the parameter t_0 are characteristics of the action of time on the causal connection. The parameter t_0, setting the force rotation phase, most likely should not manifest itself in macroscopic experiments (in a similar way phases of thermal oscillations of atoms fail to affect the macroscopic properties of bodies). Thus only two quantities: φ and ω_x may be regarded as essential characteristics of the action of time.

Assume that these quantities are related by a dependence close to

$$\omega = \omega_0 \tan\varphi, \tag{3.4}$$

where $\omega = |\omega_x| = |\vec{\omega}|$ is the absolute value of the angular velocity pseudovector $\vec{\omega}$; ω_0 is a constant of frequency dimension. Then at $\varphi = 0$ we obtain the case studied by theoretical mechanics, with the system being purely determinate. On the contrary, at $\varphi = \pi/2$ the causal action completely disappears and the system becomes absolutely indeterminate (the latter follows from the fact that at $\varphi = \pi/2$ the forces $\vec{F}_c$ and $\vec{F}_e$ are directed perpendicular to the straight line CE and rotate about it infinitely rapidly and therefore their time averages over any time interval turn out to be exactly zero). The existence of the two limiting states of a system, one strictly determinate and another absolutely indeterminate, is in complete agreement with the ideas of causal mechanics.

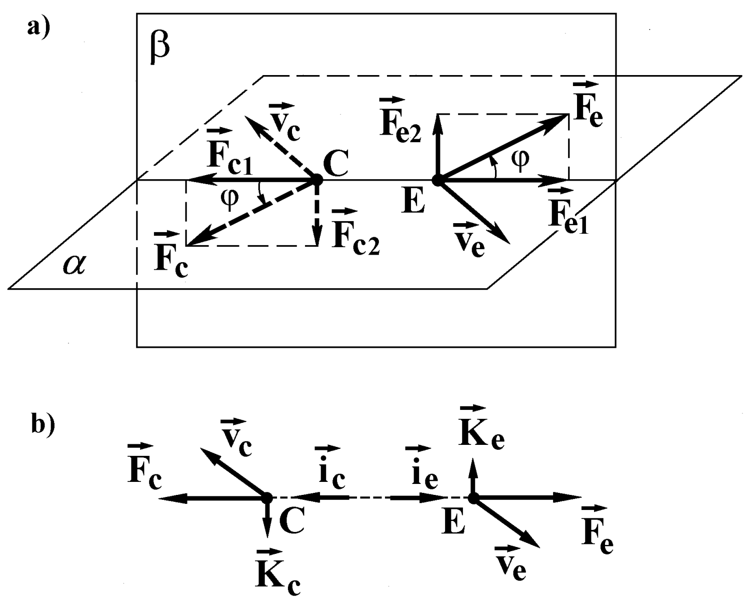

Fig.5. One more possible action of time on a causal link:
a —the deflection of the forces $\vec{F}_c$ and $\vec{F}_e$ from the straight line CE in the plane β by the angle φ depending on the relative velocity $\vec{v}_e$; b — the appearance of the additional forces $\vec{K}_c$ and $\vec{K}_e$ described by the expressions (3.6) and (3.7) or (3.9) and (3.10); $\vec{K}_c = -\vec{K}_e$; $\vec{v}_e$ — velocity of the effect point E with respect to the cause point C; $\vec{v}_c$ — velocity of the point C with respect to the point E, $\vec{v}_c = -\vec{v}_e$; $\vec{F}_{e1}$, $\vec{F}_{e2}$, $\vec{F}_{c1}$, $\vec{F}_{c2}$ — the components of the action ($\vec{F}_e$) and reaction ($\vec{F}_c$) forces directed along the line CE and perpendicular to it; α —the plane containing the relative velocity vector $\vec{v}_e$ and the straight line CE; β — the plane perpendicular to α and containing the line CE; $\vec{i}_e$ and $\vec{i}_c$ — the unit vectors lying at the line CE and directed from the point C to the point E and from E to C respectively; $\vec{i}_c = -\vec{i}_e$.

Version 2. Let the forces $\vec{F}_c$ and $\vec{F}_e$ deviate from the straight line CE as follows. If the relative motion velocity of the cause C and the effect E is directed along the straight line CE or equal to zero, a deviation is absent. If the relative velocity of the points C and E is directed at a certain angle to the straight line CE, there occurs a deviation of the forces in the plane perpendicular to another plane containing the vector of relative velocity and the straight line CE. As this takes place, the forces $\vec{F}_c$ and $\vec{F}_e$ deflect from the straight line CE in opposite directions by the same angle, as we have agreed before (Fig.5a).

One of the two possible directions of force deflection in the above plane can be defined for each element of the cause-and-effect link in the following way. Consider three vectors: (i) the velocity with which the element under consideration moves with respect to the other one, (ii) the component of the force acting on it, directed along the straight line CE, and (iii) the component of the same force directed perpendicular to the line CE. Let us ascribe numbers to these vectors in the same succession as they are listed and assume that a deflection of the force from the straight line occurs in such a direction that the above ordered triad of vectors form a left frame for the cause point and a right one for the effect point. We shall assume that the force deflection angle from the straight line CE depends on the relative motion velocity of the cause and the effect in such a way that it vanishes when the relative velocity vector direction approaches that of the line CE.

Let us consider this version in more detail for the case of small φ. In this case the force deflection from the line CE may be regarded as a consequence of action of the small additional forces $\vec{K}_c$ and $\vec{K}_e$ directed perpendicular to the line CE and connected with the angle φ by the relations

$$|\vec{K}_c| = |\vec{F}_c| \tan\varphi; \quad |\vec{K}_e| = |\vec{F}_e| \tan\varphi \tag{3.5}$$

(Fig.5b). We shall assume that the additional forces are described by the expressions

$$\vec{K}_e = \frac{1}{c_2} \vec{v}_e \times \vec{F}_e \ ; \tag{3.6}$$

$$\vec{K}_c = -\frac{1}{c_2} \vec{v}_c \times \vec{F}_c \ , \tag{3.7}$$

where the forces of action ($\vec{F}_e$) and reaction ($\vec{F}_c$) are directed along the line CE; $\vec{v}_e$ is the velocity of the effect point E with respect to the cause point C; $\vec{v}_c = -\vec{v}_e$; c_2 is a pseudo-scalar parameter of velocity dimension, $c_2 > 0$ in a right-handed coordinate frame (the *pseudo*scalarity of c_2 is required to compensate the *pseudo*vector nature of the vector product). From $\vec{F}_c = -\vec{F}_e$ and $\vec{v}_c = -\vec{v}_e$ it follows that $\vec{K}_c = -\vec{K}_e$, as expected. Since we are considering the case $\varphi \ll 1$, one can write with (3.5), (3.6):

$$\varphi \approx \tan\varphi = \frac{|\vec{K}_e|}{|\vec{F}_e|} = \frac{1}{|c_2|} |\vec{v}_e| \sin(\vec{v}_e \,\hat{,}\, \vec{F}_e), \tag{3.8}$$

therefore the condition $|\vec{v}_e| \sin(\vec{v}_e \,\hat{,}\, \vec{F}_e) \ll |c_2|$ should be satisfied. For simplicity we shall assume that $|\vec{v}_e| \ll |c_2|$. We shall discuss Eqs. (3.6), (3.7) below, after describing the third possible version of the action of time on the causal connection.

Version 3. Assume that the forces $\vec{F}_c$ and $\vec{F}_e$ deflect from the straight line CE in the same way as in Version 2 with the only exception: the deflection direction is determined by another ordered triad of vectors. Namely, let us take the following three vectors: (i) that of relative velocity of the element under consideration of the cause-and-effect link; (ii) the unit vector lying on the straight line CE and pointed towards the given element (off the other); (iii) the component of the force acting on the given element, directed perpendicular to the line CE. (In Version 2 the force component directed along the straight line CE was taken as the second vector.) Assume that the deflection of the force from the straight line CE occurs in such a way that the above three vectors, numbered in the above order, form a left frame for the cause point and a right one for the effect point. The force deflection angle φ is assumed to be the same as in Version 2.

For small φ one can again replace the deflections of the forces $\vec{F}_c$ and $\vec{F}_e$ from the line CE by adding small additional forces $\vec{K}_c$ and $\vec{K}_e$ perpendicular to this line and satisfying the relations (3.5). We shall assume that these forces are described by the expressions

$$\vec{K}_e = \frac{1}{c_2} F \vec{v}_e \times \vec{i}_e \;; \tag{3.9}$$

$$\vec{K}_c = -\frac{1}{c_2} F \vec{v}_c \times \vec{i}_c \;, \tag{3.10}$$

where $F = |\vec{F}_e| = |\vec{F}_c|$; $\vec{i}_e$ and $\vec{i}_c$ are the unit vectors lying on the straight line CE, so that $\vec{i}_e$ is drawn from the point C towards the point E, and $\vec{i}_c$ is drawn from the point E towards the point C ($\vec{i}_c = -\vec{i}_e$); the other notations are the same as in Eqs. (3.6) and (3.7) (see Fig.5b). Here, as well as in Version 2, we assume that the condition $|\vec{v}_e| << |c_2|$ is fulfilled.

Now let us consider a particular case. Let the cause point C be at rest in a certain inertial frame of reference, and the effect point E revolve uniformly about it along a circle centered at the point C. In this case the relative velocity $\vec{v}_e$ is perpendicular to the straight line CE and directed along a tangent to the circle, therefore Eqs. (3.9), (3.10) can be transformed to yield

$$\vec{K}_e = \frac{v}{c_2} F \vec{l} \;; \tag{3.11}$$

$$\vec{K}_c = -\frac{v}{c_2} F \vec{l} \;, \tag{3.12}$$

where $v = |\vec{v}_e| = |\vec{v}_c|$; $\vec{l}$ is a unit pseudovector perpendicular to the vectors $\vec{v}_e$ and $\vec{i}_e$ and pointed in the same direction as the pseudovector $\vec{v}_e \times \vec{i}_e$. Equations (3.11) and (3.12) are in agreement with those for the additional forces in causal mechanics (Kozyrev 1991). It is by similarity with the latter that we introduced the notation c_2 for the parameter entering in the right-hand sides of our formula. Note that if the cause-effect interaction is of repulsive nature, then $\vec{F}_e = F\vec{i}_e$, $\vec{F}_c = F\vec{i}_c$, and Eqs. (3.6) and (3.7) from Version 2 acquire the form (3.9), (3.10). Therefore in this particular case they can be converted to (3.11), (3.12) as well. Thus, Versions 2 and 3 proposed for the action of time on the cause-and-effect connection may be regarded as possible immediate generalizations of the corresponding propositions of causal mechanics.

It should be noted that the difference between Versions 2 and 3 manifests itself most noticeably in the case of a sign-variable interaction between the cause and the effect: as signs of the forces $\vec{F}_e$ and $\vec{F}_c$ change, the additional forces $\vec{K}_e$ and $\vec{K}_c$ in Version 2 change signs as well, whereas in Version 3 they remain unchanged. Also note that, strictly speaking, the appearance of additional forces of a certain magnitude and a deflection of the "dassical" forces by the angle determined by Eqs. (3.5), are not identical results. However, for additional forces much less in magnitude than the "dassical" ones these results differ by second-order small quantities and are indistinguishable within the measurement accuracy achieved in N.A.Kozyrev's experiments.

A distinctive feature of the additional forces $\vec{K}_e$ and $\vec{K}_c$ introduced in Versions 2 and 3, is that *in total they do not perform work over the cause-and-effect link*.

Indeed, the total increment of work ΔA of these forces for a short time interval Δt amounts to

$$\Delta A = \vec{K}_e \cdot \vec{u}_e \Delta t + \vec{K}_c \cdot \vec{u}_c \Delta t \,, \tag{3.13}$$

where $\vec{u}_e$ and $\vec{u}_c$ are the effect and cause velocities with respect to the inertial frame of reference under consideration. Taking into account that $\vec{K}_c = -\vec{K}_e$ and that the effect moves with respect to the cause with the velocity $\vec{v}_e = \vec{u}_e - \vec{u}_c$, we obtain from (3.13):

$$\Delta A = \vec{K}_e \cdot (\vec{u}_e - \vec{u}_c)\Delta t = \vec{K}_e \cdot \vec{v}_e \Delta t \,.$$

Since the additional force $\vec{K}_e$, according to Eqs. (3.6), (3.9), is perpendicular to the velocity vector $\vec{v}_e$, we obtain finally that $\Delta A = 0$.

This result is of fundamental significance. It means that no additional expenses of work are required to realize actions on the cause-and-effect link described in Versions 2 and 3. The system energy also remains unchanged under such an action. Also note that since the principal vector of the additional forces is zero, $\vec{K}_e + \vec{K}_c = \vec{0}$, the total momentum of the system remains unchanged. At the same time, this action can change the angular momentum of the system and the trajectories of the cause-and-effect link elements. Probably it is just the version of the action of time on causal connections to which N.A.Kozyrev inclined as his ideas were developed. In his first publications on causal mechanics he wrote that time was able to augment the energy of a system, while in more recent papers he asserted that time, via its active physical properties, increases the order of matter, preventing (to some extent) an increase of entropy in a system, i.e., it acts as a source of negentropy in our World.

Thus we have considered three possible versions of force vector deflection from the straight line connecting the interacting points. This deviation cannot be explained within the framework of classical mechanics by the properties of the cause-and-effect link itself due to its symmetry. Material points in classical mechanics have no internal structure, hence their symmetry coincides with that of a geometric point. That implies that among the elements of symmetry of a cause-and-effect link there is an infinite order rotation axis passing through the cause and effect points, and mirror symmetry planes containing the rotation axis. With these elements of symmetry available, no internal cause is able to deviate the interaction force from the rotation axis in some direction (as it is the case in Versions 2 and 3) or lead to its deviation and rotation in a certain direction (as in Version 1). Hence, from the viewpoint of classical mechanics, a deviation like those described could result only from causes external with respect to our cause-and-effect link.

The above three versions of the action of time on a cause-and-effect link are, of course, not the only possible ones. However, which of these or other possible versions reflects the reality appropriately, can be decided only from the results of special experiments.

As seen from the present section, in classical mechanics itself there exists a possibility of force deviation from the straight line connecting the interacting points (there is, however, no physical reason for such a deviation in a certain direction). Therefore Kozyrev's causal mechanics may be regarded as a natural development of Newton's classical mechanics.

4. On inaccuracy of force representation in classical mechanics

By the fundamental postulates of causal mechanics, a cause and an effect are separated by arbitrarily small but nonzero space (δx) and time (δt) differences, with the time difference being of a definite sign since an effect comes after a cause. N.A.Kozyrev has called the ratio of these quantities the *course of time* c_2:

$$c_2 = \delta x / \delta t \, . \tag{4.1}$$

The proposition that a cause and an effect cannot be spatially superimposed is used in classical mechanics as well. This proposition follows from Newton's third law according to which the forces of action and reaction are applied to different bodies, meaning that there necessarily exists a nonzero spacing between the force application points. At the same time classical mechanics neglects the time difference between the cause and the effect. It is also apparent in Newton's third law, where the forces applied to the cause and the effect act at the same instant. Thus one can say that classical mechanics is a degenerate case of causal mechanics corresponding to the following values of quantities: $\delta x \neq 0$, $\delta t = 0$ and $c_2 = \infty$ (Kozyrev 1991).

Neglecting the time difference between the cause and the effect leads to inaccuracy of setting the directions and magnitudes of forces in classical mechanics. Let us show that.

Assume that the four-dimensional proper Euclidean space is a geometric image of space and time (which is known not to be contrary to classical mechanics). Since the four coordinates in this space should be measured with the same units, we assume, by analogy with the theory of relativity, that the time coordinate is ct, where c is the velocity of light in vacuum.

In the present section we shall interpret the quantities $|\delta x|$ and $|\delta t|$ differently from what was done in Section 1. These quantities will be considered to be determinate, i.e. taking quite definite values for specific cause-and-effect links, which may, however, be different for different links. It is this treatment of the above quantities that was used in N.A.Kozyrev's papers (which is to be judged only from the context, since this issue was not discussed in detail by Kozyrev (1991)). We shall assume that Kozyrev's law

$$|c_2| = |\delta x| / |\delta t| \equiv \alpha c \qquad (4.2)$$

is valid, where α is the fine structure constant ($\alpha \approx 1/137$). It should be emphasized that in what follows our attention will be focused on "classical" forces, while the addi tional ones, considered in Section 3, will be discussed only at the end of the present section.

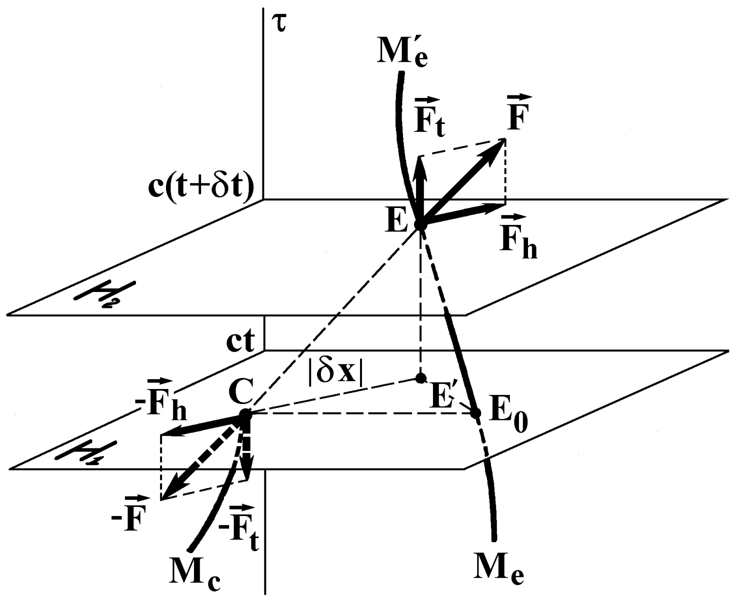

Fig.6. A cause C and an effect E in the process of a causal interaction: $\vec{F}$ —the cause-effect interaction force; $\vec{F}_t$ —the temporal component of the force $\vec{F}$; $\vec{F}_h$ —the component of $\vec{F}$ along the hyperplane of simultaneous events; $|\delta x|$, $|\delta t|$ —the spatial and temporal intervals between the cause and the effect at a causal interaction; τ —the time axis; H_1, H_2 —the hyperplanes of simultaneous events, passing through the cause and effect points, respectively; M_cC —the world line of the cause (only its part up to the instant t is shown); $M_eM'_e$ —the world line of the effect; E_0 —the intersection point of the hyperplane H_1 and the world line of the effect; E' —the projection of the effect point E on H_1; c —the velocity of light in vacuum; it has been taken into account that the effect occurs later than the cause; the hyperplanes H_1 and H_2 are drawn with the dimension reduced by one.

The fact that the cause and the effect manifest themselves at different instants means that they belong to different hyperplanes of simultaneous events (Fig.6). This raises the question: "Where are the forces, applied to the cause and the effect, directed: do they lie in the corresponding hyperplanes of simultaneous events or are they directed along the straight line connecting the cause and effect points?" Classical mechanics does not allow one to make a choice between these possibilities. Therefore we make use of considerations of symmetry. Since the cause-and-effect link incorporates a rotation axis, passing through its points, as an element of symmetry, it is natural to expect that the force system connected with it, has the same symmetry. This gives a ground to believe that the interaction forces are directed along the straight line connecting the cause and the effect, as shown in Fig.6. Such an orientation of forces fits the relativistic symmetry of space and time as well. (Note that this consideration does not apply to the additional forces of Section 3, since the symmetry of the latter is determined by the properties of not only the cause-and-effect link but those of time as well.)

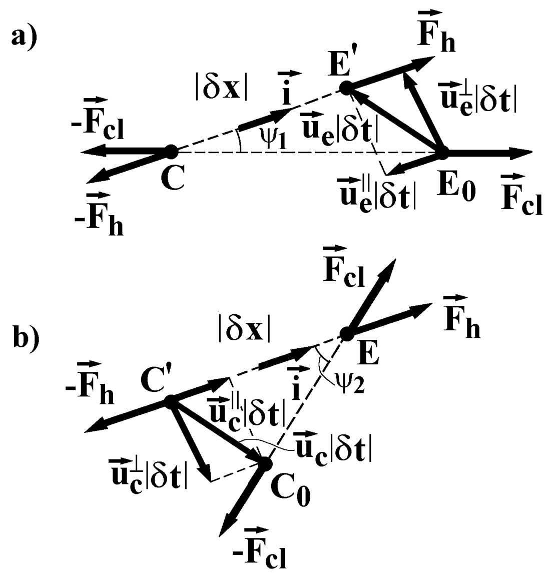

Fig.7. The projections of a cause-and-effect link onto hyperplanes of simultaneous events passing through the cause point C (a) and the effect point E (b): $\vec{F}_h$ — the cause-effect interaction force component directed along the hyperplane of simultaneous events; $\vec{F}_{cl}$ — the force considered in classical mechanics; ψ_1, ψ_2 — the angles between the forces $\vec{F}_h$ and $\vec{F}_{cl}$; E_0 — the intersection point between the world line of the effect and the hyperplane of simultaneous events passing through the cause point; E′ — the projection of the effect point E onto the same hyperplane; C_0 — the intersection point of the cause world line and the hyperplane of simultaneous events passing through the effect point; C′ — the projection of the cause point C onto the same hyperplane; $|\delta x|$, $|\delta t|$ — the cause-effect spatial and temporal intervals during the causal interaction; $\vec{u}_e, \vec{u}_e^{\parallel}, \vec{u}_e^{\perp}$ — the effect velocity vector and its components parallel and perpendicular to the force $\vec{F}_h$; $\vec{u}_c, \vec{u}_c^{\parallel}, \vec{u}_c^{\perp}$ — the same for the cause; $\vec{i}$ — the unit vector along the line of action of the force $\vec{F}_h$, directed from the point C (or C′) to the point E′ (or E).

Being directed as described, the interaction forces have a nonzero temporal component neglected by classical mechanics. Let us find a relation between this component and that lying within the hyperplane of simultaneous events. As a straight line, being projected onto a hyperplane, passes to another straight line, the interaction force vector $\vec{F}$ and both of its components lie in the (two-dimensional) plane passing through the three points: the cause point C, the effect point E and the point E′ (where E′ is the projection of the point E onto the hyperplane of simultaneous events corresponding to the point C). One of the components of the vector $\vec{F}$ is perpendicular and the other is parallel to the segment CE′. Taking this fact into account, one can see from Fig.6 that the component $\vec{F}_t$ directed along the time axis and the component $\vec{F}_h$ directed along the hyperplane of simultaneous events are connected by the relation

$$|\vec{F}_t| / |\vec{F}_h| = c|\delta t| / |\delta x| .$$

Hence, using the law (4.2), we find

$$|\vec{F}_t| = \frac{c}{|c_2|}|\vec{F}_h| = \frac{1}{\alpha}|\vec{F}_h| \approx 137|\vec{F}_h| . \quad (4.3)$$

Thus generally the condition $\delta t \neq 0$ may result in the appearance of a time component of the interaction force. One of the inaccuracies of handling forces in classical mechanics lies just in neglecting that possibility.

Needless to say that the assertion of interaction forces being directed along the line connecting the cause and the effect is no more than a hypothesis. Other versions are also possible. For instance, if, as it is done in relativity theory, one determines the force as a derivative of the momentum with respect to time, it will necessarily lie in the hyperplane of simultaneous events, since the momentum vector lies there. At the same time, as long as the question of a real direction of the interaction forces has not been conclusively solved, it is necessary to take into account the possibility that the time component be present in the forces.

In classical mechanics an inaccuracy of force representation is also present due to neglecting a mutual displacement of the cause and the effect taking place during the time interval δt. Let us estimate this inaccuracy.

In classical mechanics it is assumed that the cause and the effect happen at the same instant. This means that the interaction forces are applied at the points of the world lines of the cause and the effect located at the same hyperplane of simultaneous events. If $\delta t \neq 0$, then such a hyperplane may be arbitrarily chosen among the hyperplanes placed between those of the cause and the effect (both are shown in Fig.6).

Let us analyse the extreme situations when just these two surfaces serve as the hyperplane considered in classical mechanics (Fig.7). The figure demonstrates that in these two cases the segment connecting simultaneous events of the world lines of the cause and the effect is directed differently and varies in length due to their mutual displacement (these are the segments CE_0 and C_0E in Figs.7a and 7b, respectively). The interaction force considered in classical mechanics is directed just along this segment and is unambiguously determined by its length. In the figure it is denoted by $\vec{F}_{cl}$. At the same time the component $\vec{F}_h$ of the real interaction force $\vec{F}$ has another direction, namely, along the segment connecting the projections of the cause point C and the effect point E onto the hyperplane of simultaneous events (these are CE' and $C'E$ in Figs.7a and 7b, respectively). Note that the line of action of the component $\vec{F}_h$ is the same for any direction of the interaction force $\vec{F}$ in the plane CEE' (see Fig.6), in particular, when a time component of the force is absent, i.e., for $\vec{F} = \vec{F}_h$. It should be noted as well that the (two-dimensional) planes where the prototypes of the system of vectors depicted in Figs.7a and 7b lie, may be non-coplanar in the four-dimensional space; however, the straight lines belonging to those planes and labelled in the figures by the unit vector $\vec{i}$, are mutually collinear.

Let us first estimate the direction inaccuracy of the force $\vec{F}_{cl}$, neglecting the inaccuracy of its magnitude.

Assume that the accelerations of the interacting points are so small that the world line segments passed by them for the time interval δt, are close to rectilinear. Then the projections of these segments (i.e., the lines E_0E' and C'_0C in Fig.7) are close to rectilinear as well. Hence it is easily assured that the angles ψ_1 and ψ_2 between the forces $\vec{F}_h$ and $\vec{F}_{cl}$ are expressed as follows:

$$\tan\psi_1 = \frac{|\vec{u}_e^{\perp}||\delta t|}{|\delta x| - \vec{i}\cdot\vec{u}_e^{\parallel}|\delta t|} = \frac{|\vec{u}_e^{\perp}|}{|c_2| - \vec{i}\cdot\vec{u}_e^{\parallel}};$$

$$\tan\psi_2 = \frac{|\vec{u}_c^{\perp}||\delta t|}{|\delta x| - \vec{i}\cdot\vec{u}_c^{\parallel}|\delta t|} = \frac{|\vec{u}_c^{\perp}|}{|c_2| - \vec{i}\cdot\vec{u}_c^{\parallel}}, \qquad (4.4)$$

where $\vec{u}_e^{\perp}$ and $\vec{u}_e^{\parallel}$ are the components of the effect motion velocity, perpendicular and parallel to the force $\vec{F}_h$, respectively; $\vec{u}_c^{\perp}$ and $\vec{u}_c^{\parallel}$ are the same for the cause; $\vec{i}$ is the unit vector lying on the line of action of the force $\vec{F}_h$ and directed from the point C (or its projection C') to the point E' (or E); here the law (4.2) has been used.

We shall assume that the velocities of motion of the effect $\vec{u}_e$ and the cause $\vec{u}_c$ are small compared with the constant c_2: $|\vec{u}_e| << |c_2|$, $|\vec{u}_c| << |c_2|$. Then, based on (4.4), one can write (in the linear approximation in $|\vec{u}_e|/|c_2|$, $|\vec{u}_c|/|c_2|$)

$$\psi_1 \approx |\vec{u}_e^{\perp}|/|c_2| << 1; \quad \psi_2 \approx |\vec{u}_c^{\perp}|/|c_2| << 1. \tag{4.5}$$

In this case the difference $\vec{F}_h - \vec{F}_{cl}$ is approximately described by the following formula for the two cases under consideration, assuming that the lengths of the vectors $\vec{F}_h$ and $\vec{F}_{cl}$ are nearly equal (see Fig.7):

$$\vec{F}_h - \vec{F}_{cl} \approx \gamma \frac{\vec{u}_e^{\perp}}{|\vec{u}_e^{\perp}|} |\vec{F}_h| \psi_1 \approx \gamma \frac{\vec{u}_e^{\perp}}{|c_2|} |\vec{F}_h|;$$

$$\vec{F}_h - \vec{F}_{cl} \approx \gamma \frac{\vec{u}_c^{\perp}}{|\vec{u}_c^{\perp}|} |\vec{F}_h| \psi_2 \approx \gamma \frac{\vec{u}_c^{\perp}}{|c_2|} |\vec{F}_h|, \tag{4.6}$$

where $\gamma = \mathrm{sign}(\vec{F}_h \cdot \vec{i})$. The coefficient γ sets the sign of the expression which depends on whether the cause and the effect attract ($\vec{F}_h \cdot \vec{i} < 0$) or repel ($\vec{F}_h \cdot \vec{i} > 0$) each other; the factors $\vec{u}_e^{\perp}/|\vec{u}_e^{\perp}|$ and $\vec{u}_c^{\perp}/|\vec{u}_c^{\perp}|$ serve as a direction unit vector setting the direction of the force $\vec{F}_h - \vec{F}_{cl}$.

From the relations (4.6) it follows that the inaccuracy of the action direction of the force $\vec{F}_{cl}$ can be compensated by adding to it an additional force $\vec{F}_{\perp}$ equal on the average to

$$\vec{F}_{\perp} \approx \gamma \frac{\vec{u}_e^{\perp} + \vec{u}_c^{\perp}}{2|c_2|} |\vec{F}_h|. \tag{4.7}$$

The same relations imply that the extreme positions of the force $\vec{F}_{cl}$ depicted in Figs. 7a and 7b differ by the value $\Delta\vec{F}_{cl}^{\perp}$ equal to

$$\Delta\vec{F}_{cl}^{\perp} \approx \gamma \frac{\vec{v}_e^{\perp}}{|c_2|} |\vec{F}_h|, \tag{4.8}$$

where $\vec{v}_e^{\perp}$ is the component of the motion velocity of the effect $\vec{v}_e$ with respect to the cause ($\vec{v}_e = \vec{u}_e - \vec{u}_c$) perpendicular to the force $\vec{F}_h$. It should be noted that the quantity $\Delta\vec{F}_{cl}^{\perp}$ is of invariant nature since it is determined by the relative velocity of motion of the cause and the effect, whereas the quantity $\vec{F}_{\perp}$, being connected with the absolute velocity values, depends on the choice of the frame of reference and hence is not invariant.

Let us now estimate the inaccuracy of setting the magnitude of the force $\vec{F}_{cl}$ (neglecting the inaccuracy of its direction).

Consider a typical interaction law such that

$$\left|\vec{F}_{cl}\right| = \frac{f}{r^2}, \tag{4.9}$$

where f denotes all the relevant quantities except the distance; r is the distance between the interacting material points. By the postulates of causal mechanics, a spacing between the cause and the effect in interaction is $|\delta x|$. Meanwhile in the two cases depicted in Fig.7 the spacings r_1 and r_2 between the application points of the "classical" forces (i.e. the lengths of the segments CE_0 and C_0E) are other than $|\delta x|$ and amount to

$$r_1 = \frac{|\delta x| - \vec{i}\cdot\vec{u}_e^{\parallel}\,|\delta t|}{\cos\psi_1}; \quad r_2 = \frac{|\delta x| - \vec{i}\cdot\vec{u}_c^{\parallel}\,|\delta t|}{\cos\psi_2}. \tag{4.10}$$

In the case of $|\vec{u}_e| \ll |c_2|$ and $|\vec{u}_c| \ll |c_2|$, as follows from Eqs. (4.5), the approximate equalities $\cos\psi_1 \approx 1$ and $\cos\psi_2 \approx 1$ are valid (in the linear approximation in $|\vec{u}_e|/|c_2|$ and $|\vec{u}_c|/|c_2|$). Based on the latter and the law (4.2), we obtain from (4.10) the following values of r_1 and r_2:

$$r_1 \approx |\delta x|\left(1 - \frac{\vec{i}\cdot\vec{u}_e^{\parallel}}{|c_2|}\right); \quad r_2 \approx |\delta x|\left(1 - \frac{\vec{i}\cdot\vec{u}_c^{\parallel}}{|c_2|}\right). \tag{4.11}$$

A substitution of these distance values into (4.9) gives the following values for the force magnitudes:

$$\left|\vec{F}_{cl}\right| \approx \frac{f}{|\delta x|^2\left(1 - \frac{\vec{i}\cdot\vec{u}_e^{\parallel}}{|c_2|}\right)^2} \approx F\left(1 + 2\frac{\vec{i}\cdot\vec{u}_e^{\parallel}}{|c_2|}\right);$$

$$\left|\vec{F}_{cl}\right| \approx \frac{f}{|\delta x|^2\left(1 - \frac{\vec{i}\cdot\vec{u}_c^{\parallel}}{|c_2|}\right)^2} \approx F\left(1 + 2\frac{\vec{i}\cdot\vec{u}_c^{\parallel}}{|c_2|}\right), \tag{4.12}$$

where $F = f/|\delta x|^2$ is the real value of the "classical" interaction force magnitude.

From Eqs.(4.12) it follows that the inaccuracy of setting the magnitude of the force $\vec{F}_{cl}$ may be compensated by adding to it a supplementary force $\vec{F}_{\parallel}$ equal on the average to

$$\vec{F}_{\parallel} \approx -\gamma \frac{\vec{u}_e^{\parallel} + \vec{u}_c^{\parallel}}{|c_2|} F \ , \qquad (4.13)$$

where it has been taken into account that the vectors $\vec{i}$, $\vec{u}_e^{\parallel}$, $\vec{u}_c^{\parallel}$ and $\vec{F}_h$ are mutually collinear and approximately parallel to the vector $\vec{F}_{cl}$. From formula (4.12) it follows as well that the range of magnitudes of the force $\vec{F}_{cl}$ in these two cases is such that the corresponding differential force $\Delta \vec{F}_{cl}^{\parallel}$ is

$$\Delta \vec{F}_{cl}^{\parallel} \approx -\gamma \frac{2\vec{v}_e^{\parallel}}{|c_2|} F \ , \qquad (4.14)$$

where $\vec{v}_e^{\parallel}$ is the component of the relative velocity of the effect $\vec{v}_e$ parallel to the force $\vec{F}_h$. Here, as before, the force $\Delta \vec{F}_{cl}^{\parallel}$ is an invariant quantity, while $\vec{F}_{\parallel}$ is not.

Using (4.7), (4.8), (4.13) and (4.14) in practice, it is convenient to express the forces they set in terms of the mean value of the "classical" force. In the following just this mean value will be denoted by $\vec{F}_{cl}$. Since these forces are small compared with $\vec{F}_{cl}$, the formula obtained will remain valid (in the linear approximation in $|\vec{u}_e|/|c_2|$ and $|\vec{u}_c|/|c_2|$, as considered), if one substitutes the real forces in them by their approximate "classical" value and, moreover, assumes that the velocity components denoted by the symbols $\perp$ and $\parallel$ are directed in perpendicular and parallel to the force $\vec{F}_{cl}$ but not to the force $\vec{F}_h$. Performing these changes, we conclude on the basis of (4.8) and (4.14) that the difference between the extreme values of the "classical" force can be presented in the form of a sum of two components, of which the first one is perpendicular and the second one parallel to the force $\vec{F}_{cl}$:

$$\Delta \vec{F}_{cl}^{\perp} \approx \gamma \frac{\vec{v}_e^{\perp}}{|c_2|} F \, ; \qquad (4.15)$$

$$\Delta \vec{F}_{cl}^{\parallel} \approx -\gamma \frac{2\vec{v}_e^{\parallel}}{|c_2|} F \ , \qquad (4.16)$$

where $\gamma = \mathrm{sign}(\vec{F}_{cl} \cdot \vec{i})$; $F = |\vec{F}_{cl}|$. By (4.7) and (4.13), we arrive at the conclusion that the supplementary forces to be added to the "classical" force $\vec{F}_{cl}$ to compensate the inaccuracies of its direction and magnitude, are of the form

$$\vec{F}_{\perp} \approx \gamma \frac{\vec{u}_e^{\perp} + \vec{u}_c^{\perp}}{2|c_2|} F \, ; \qquad (4.17)$$

$$\vec{F}_{\parallel} \approx -\gamma \frac{\vec{u}_e^{\parallel} + \vec{u}_c^{\parallel}}{|c_2|} F \ , \tag{4.18}$$

the first one of these forces being perpendicular and the second one parallel to the force $\vec{F}_{cl}$. Recall that the interaction law (4.9) was used in deriving Eqs. (4.16) and (4.18).

Let us unify all that with the temporal component of the interaction force, as it has been discussed earlier. From (4.3) it is easily obtained that

$$\vec{F}_t = \gamma \frac{\vec{V}}{|c_2|} |\vec{F}_h| \approx \gamma \frac{\vec{V}}{|c_2|} F \ , \tag{4.19}$$

where $\vec{V}$ is a "velocity" of motion of our World along the time axis ($\vec{V}$ is parallel to the time axis, directed from the past to the future and has the magnitude c: $|\vec{V}| = c$); here it is taken into account that the vector $\vec{F}_t$ is pointed in the same direction as the vector $\vec{V}$ in the case of repulsion and oppositely in the case of attraction (see Fig.6).

Thus in classical mechanics the interaction force proves to be inaccurately fixed due to a neglect of time difference in the instants of appearance of the cause and the effect. It has an error in the components values along the three mutually perpendicular directions: the time axis and two directions lying in the hyperplane of simultaneous events — along the force itself and perpendicular to it.

In Section 3 one more inaccuracy of the "classical" force, the one due to a specific action of time, was discussed. Let us write down all the four additives compensating the inaccuracies of the "classical" forces as applied to a particular case of the cause point being at rest ($\vec{u}_c = \vec{0}$). Using Eqs. (3.11), (4.17) - (4.19), we obtain

$$\vec{K}_e = \frac{|\vec{v}_e^{\perp}|}{c_2} \vec{l} F; \ \vec{F}_{\perp} \approx \gamma \frac{\vec{v}_e^{\perp}}{2|c_2|} F; \ \vec{F}_{\parallel} \approx -\gamma \frac{\vec{v}_e^{\parallel}}{|c_2|} F; \ \vec{F}_t \approx \gamma \frac{\vec{V}}{|c_2|} F. \tag{4.20}$$

Here we have taken into account the following: (a) the cause-and-effect link as considered in Section 3 is actually a pair of simultaneous points on the world lines of the cause and the effect, therefore the points C and E and the force $\vec{F}_e$ from Section 3 are, in fact, the points C and E_0 (or C_0 and E) and the force $\vec{F}_{cl}$ from the present section, respectively (cf. Figs.3 - 5 with Figs.6, 7); (b) Eq. (3.11) written for the case of $\vec{u}_c = \vec{0}$, $\vec{u}_e^{\parallel} = \vec{0}$ remains valid for $\vec{u}_e^{\parallel} \neq \vec{0}$ as well if one substitutes the quantity v (equal to $|\vec{v}_e|$) by $|\vec{v}_e^{\perp}|$ (since the component $\vec{v}_e^{\parallel}$ does not contribute to the force $\vec{K}_e$ according to (3.6) and (3.9)); (c) $\vec{u}_e = \vec{v}_e$ for $\vec{u}_c = \vec{0}$. Note that all the four additional forces are mutually perpendicular (recall that the unit vector $\vec{l}$ is orthogonal to the vectors $\vec{v}_e$ and $\vec{F}_{cl}$).

Attention should be drawn to the fact that all the formula of (4.20) are of the same kind. All the additional forces being described by them are, first, proportional to the absolute value of the "classical" force and, second, proportional to the ratio of the corresponding velocity to the constant c_2. This gives one more, if only indirect, argument in favour of introduction of the additional force $\vec{K}_e$; at any rate in the absence of it the symmetry of the four linearly independent directions of space-time would have been violated.

Concluding the present section, we draw attention to a possibility of giving two different interpretations of the pattern depicted in Fig.6 (independent of whether or not the force $\vec{F}$ has a time component). The first interpretation is based on the conception of our World as a three-dimensional hyperplane of exactly zero thickness along the time axis. In agreement with this idea the figure under consideration is an image of two states of the World separated by a time interval δt. Besides, there occurs an interaction between the future and the past states of the World. Another interpretation is based on the assumption of our World having nonzero thickness along the time axis or, speaking in the spirit of quantum mechanics, there is a "smearing" or "uncertainty" along this axis. In this case one might say that the pattern in Fig.6 depicts two interacting material points belonging to the same state of the World but lying in its different temporal sections.

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