Magneto-Vortex Dynamo Model

in Solar convection zone.

Sergey V. Ershkov

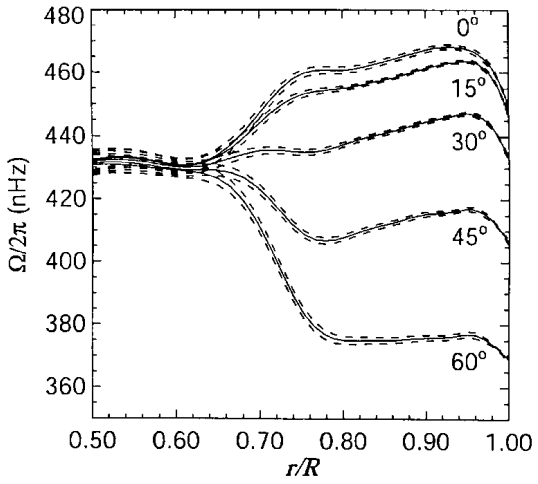
Institute for Time Nature Explorations,

M.V. Lomonosov's Moscow State University,

Leninskie gory, 1-12, Moscow 119991, Russia.

Here is presented a new magneto-vortex dynamo model for modeling & predicting of a processes in Solar plasma convection zone [1-3]. Solar convection zone is located above the level $r > 0,6 \div 0,7\ R$, where R is a Solar radius (see Pic.1 below).

A key feature of such a model is that equation of Solar plasma motion as well as equation of magnetic fields evolution - are reduced to Helmholtz's vortex equation [4], which is up-graded in according with α-effect [1-3].



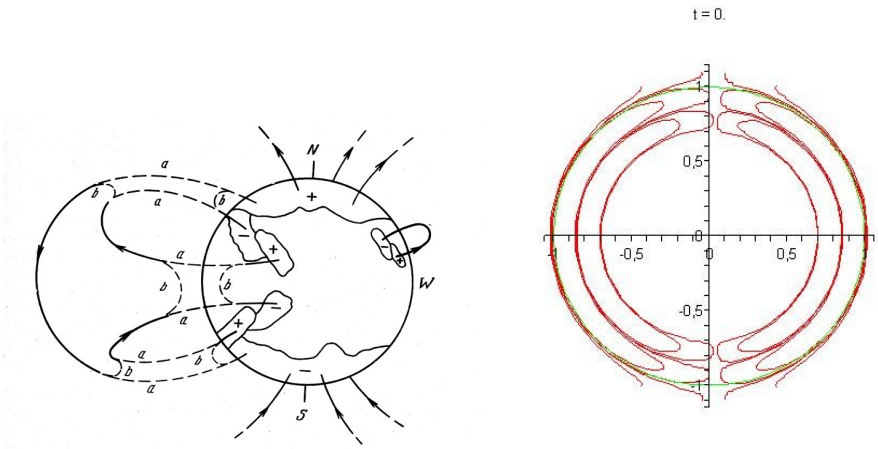
Pic.1. Distribution of Solar plasma angular velocities,

depending on radius of Solar zone [1-3].

Modeling of processes in Solar convection zone:

Below are represented a key steps of modern science scheme for evolution of Solar magnetic fields [1-3]:

1. It is the fact that a Solar *inner* poloidal magnetic field (see Pic.2) has to be located in Sun's core [3], below the Solar convection zone $r < 0,6 \div 0,7\ R$ (Pic.1):



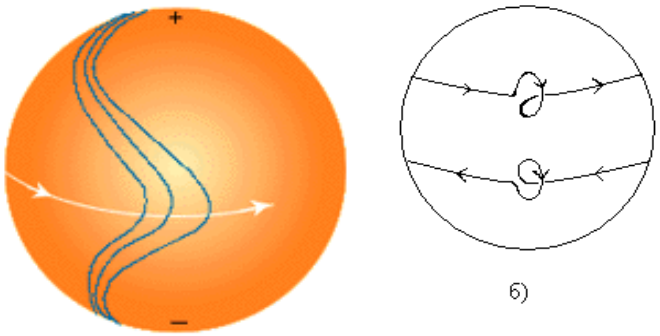
Pic.2. Solar poloidal magnetic field (*magnetic dipole*).

2. Besides, a proper scheme of 'frozen-in magnetic field' plasma motion *or opposite scheme of 'frozen-in plasma motion' magnetic field* are assumed to be actual in Solar convection zone [5]:

   - It takes place when plasma crosses magnetic field lines in its motion inside the Solar convection zone [1-3]; the main reason is *the great conductivity* of Solar plasma [2]: once intersecting a proper magnetic field line, particles of plasma will immediately be 'catching' by it, then particles are assumed to be moving only along a proper magnetic field line.

3. The next important step - is a 'differential rotation' [1-3]:

   - Shearing of poloidal magnetic field by the Sun's differential rotation in convection zone. The Sun rotates faster at the equator than the pole (20-30%) [3]. Toroidal field is produced due to this shearing by differential rotation [6] as well as due to effect of 'frozen-in magnetic field' plasma motion (*so-called ω-effect*, Pic.3):



Pic.3. Shearing of poloidal field by the Sun's differential rotation (*ω-effect*).

4. A key point is a transforming of toroidal magnetic field – back into additional poloidal magnetic field in Solar convection zone:

   - An additional field of Coriolis force initiates so-called '*α-effect*' [1-3] (Pic.3 b): vortex in plasma is moving poleward under the influence of Coriolis force near equator. Then collecting near the pole, such a vorticity forms an additional total projection on *poloidal* magnetic field due to scheme of 'frozen-in plasma motion' magnetic field.

5. Limitation of '*α-effect*':

   - We should note that there is no problem with limitation of '*α-effect*' in above scheme. An actual mechanism of 'Solar refining from extra-magnetic field' regenerates a new magnetic field due to the process of magnetic field diffusion

in plasma convection zone as well as due to magnetic field 'frozen-in Solar wind plasma' which is leaving the Sun (a very regular process).

Besides, as a result of above scheme (steps 1-4) we should obtain that a maximum of Solar activity – *strong Solar wind, extra-flares, Sunspots with strong magnetic field* - are assumed to be concentrated at the proper belt in Solar convection zone, i.e. in belt from *middle* latitudes of the Sun (*~30° latitude in respect to equator*) up to equator itself: namely such a Solar activity will be located where vorticity had begun massively to collect from equator under the influence of Coriolis force. For example, one of such axisymmetric belt phenomena is well-known *"Maunder's butterfly" diagram* [2].

Mathematical Model of Magneto-Vortex Dynamo in Solar convection zone:

In accordance with [5], we note that equation of motion for each components of plasma should be represented in form of *generalised vorticity* evolution:

$$\vec{\Omega} = \mathrm{rot}\,\vec{P}, \quad \vec{P} = m\vec{v} + e\cdot\vec{A}/c \quad \left(\vec{B} = e\cdot\mathrm{rot}\,\vec{A}/c,\ \mathrm{rot}\,\vec{v} = \vec{w}\right)$$

- here m – is the mass of particle, **v** –velocity of particle, e – it's charge, c – speed of light in vacuum, **B** – vector of local magnetic field.

When the electro-magnetic part is dominating in above expression for generalised plasma vorticity **Ω**, we obtain a super-position of magnetic field **B** (*'frozen-in plasma motion* **v***'*) & vortex field **w** = **rot v**, which is described by Helmholtz's vortex equation [4], to be generalised & up-graded with Coriolis-effect [3]:

$$\vec{w}_t = \nu\cdot\Delta\vec{w} + \mathrm{rot}\left[\vec{v}\times(\vec{B}+\vec{w})\right] + \mathrm{rot}(\alpha\vec{w}), \qquad (1)$$

$$\vec{B}\sim\vec{v} \ \Leftrightarrow\ \vec{w}_t = \nu\cdot\Delta\vec{w} + \mathrm{rot}\left[\vec{v}\times\vec{w}\right] + \mathrm{rot}(\alpha\vec{w})$$

- here ν – is a kinematic viscosity, α – some numeric coefficient.

When a vortex part **w = rot v** is dominating in above expression for generalised plasma vorticity **Ω** (*'frozen-in plasma motion* **v***'*, **w ~ v**), we obtain a proper equation for magnetic field **B** which is diffusing in Solar convection zone, such an equation is up-graded *with 'α-effect'* [3]:

$$\vec{B}_t = \mu\cdot\Delta\vec{B} + \mathrm{rot}\left[\vec{v}\times\vec{B}\right] + \mathrm{rot}\left(\alpha\vec{B}\right), \qquad (2)$$

$$\vec{w}\sim\vec{v},\ \Rightarrow\ \vec{w}_t = \nu\cdot\Delta\vec{w} + \mathrm{rot}\left[\vec{v}\times\vec{B}\right] + \mathrm{rot}\left(\alpha\vec{w}\right)$$

- here μ – is a magnetic viscosity.

Indeed, let's obtain it in details:

I. Near equator, the direction of toroidal magnetic field *coincide with direction of plasma motion* (Pic.3).

In this case, the condition of magnetic field being 'frozen-in plasma motion' means that magnetic field *is not diffusing* in plasma as well as *it is not being under the influence of 'α-effect' yet* [3], but means that magnetic field *is moving 'frozen-in plasma motion'* around the equator (*see* (2)):

$$\vec{B}\sim\vec{v}\ \ \Rightarrow\ \ \vec{B}_t = \mathrm{rot}\left[\vec{v}\times\vec{B}\right] = 0\ \ .$$

Besides, we should note that vortex field **w** = **rot v** (in expression for **Ω**) is diffusing according to (1) as well as it starts to be under the influence of Coriolis-effect (*vector multiplying is not equal to zero*), but it is not yet being under the *'α-effect'* for vortex field totally [3]:

$$\vec{w}_t = \nu\cdot\Delta\vec{w} + \mathrm{rot}\left[\vec{v}\times\vec{w}\right] \qquad (1.1)$$

II. When *'α-effect'* actually begin to arise in Solar convection zone (*at final we should have a total mechanical vorticity, to be concentrated near the ~30° of latitude in respect to equator*), it means that magnetic field begin to be 'unfrozen-in plasma motion', but mechanical vortex – opposite, to be 'frozen-in plasma motion' (*besides, 'w-effect' is already over, hence vector multiplying is equal to zero in* (1)):

$$\vec{w} \sim \vec{v},\ \mathrm{rot}\left[\vec{v}\times\vec{w}\right]=0,\quad \Rightarrow\quad \vec{w}_t = \nu\cdot\Delta\vec{w} + \mathrm{rot}\left(\alpha\vec{w}\right)\quad .$$

In this case, magnetic field should also be diffusing in plasma as well as it should be under the proper influence of Coriolis-effect or *'α-effect'* for vortex/magnetic field (*besides, 'w-effect' is already over, hence vector multiplying is equal to zero in* (2)):

$$\vec{B}_t = \mu\Delta\vec{B} + \mathrm{rot}\left(\alpha\vec{B}\right) \qquad (2.1)$$

Thus, in Solar convection zone (starting at equator – across the middle latitude – up to the pole) we observe a proper evolution of generalised vorticity **Ω**, taking into consideration the inter-changing of vortex (mechanical part of *generalised vorticity*) & magnetic field:

- on equator the magnetic field is being conserved (*'frozen-in plasma motion'*), but mechanical vortex is to be diffusing in accordance to (1.1),
- at the middle latitude vortex is being *'frozen-in plasma motion'*, but magnetic field, as well as vortex, is diffusing in accordance to (2.1).

We should also note that both of equations (2.1) - for vortex **w** as well as for magnetic field **B** (*in the Sun-spots, near ~30° of latitude*) - *are identical one to each other in mathematical sense*, but only physical essence has to be changed for our inter-changing it one to each other (*for the case of mechanical vortex/or for the case of magnetic field*).

Above equation (2.1) for each components of magnetic field **B** – is proved to be a type of inhomogeneous heat partial differential equation [7] in Cartesian coordinate system for 3-D case, with vorticity **rot** (α**B**) as a source of internal generation of magnetic field **B**.

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